

6

Triangles

Fastrack Revision

- ▶ **Similar Figures:** Two figures having the same shapes (and not necessarily the same size) are called similar figures.
- ▶ **Similar Triangles:** Two triangles are said to be similar if:
 1. their corresponding angles are equal.
 2. their corresponding sides are in the same ratio (or proportional).

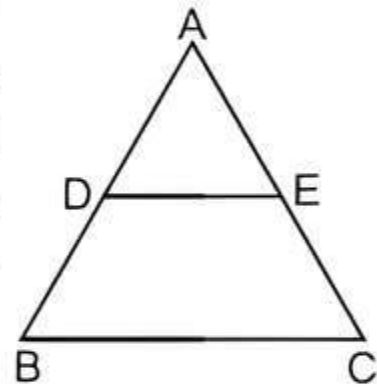
Note: Suppose $\triangle ABC$ is similar to $\triangle PQR$, we write as $\triangle ABC \sim \triangle PQR$. But we do not write as $\triangle ABC \sim \triangle QRP$ or $\triangle BAC \sim \triangle PQR$.

- ▶ **Equiangular Triangles:** If corresponding angles of two triangles are equal, then they are equiangular triangles. The ratio of any two corresponding sides of each pair in two equiangular triangles is always the same.

- ▶ **Basic Proportionality Theorem—BPT (Thales' Theorem):** In a triangle, a line drawn parallel to one side, to intersect the other two sides at distinct points, divides the two sides in the same ratio.

In $\triangle ABC$, $DE \parallel BC$, then $\frac{AD}{DB} = \frac{AE}{EC}$,

$$\frac{AD}{AB} = \frac{AE}{AC} \text{ and } \frac{AB}{DB} = \frac{AC}{EC}$$



- ▶ **Converse of Basic Proportionality Theorem:** If a line divides any two sides of a $\triangle ABC$ in the same ratio, i.e., $\frac{AD}{DB} = \frac{AE}{EC}$, then the line must be parallel to the third side, i.e., $DE \parallel BC$.

- ▶ **Criterion for Similarity of Triangles:** There are three criteria for similarity of triangles :

1. **AAA Similarity:** In two triangles, if three angles of one triangle are respectively equal to the three angles of the other triangle, then the two triangles are similar.

If two of their angles are equal, then the third angle must also be equal, because sum of angles of a triangle always make 180° . So, AA could also be called similarity.

2. **SSS Similarity:** In two triangles, if the corresponding sides are proportional, then they are similar.

Or

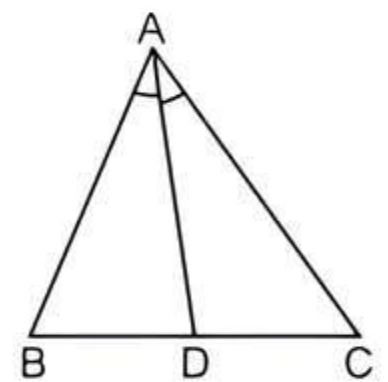
In two triangles, if sides of one triangle are proportional to the sides of the other triangle, then their corresponding angles are equal and hence the two triangles are similar.

3. **SAS Similarity:** In two triangles, if one pair of corresponding sides are proportional and the included angles are also equal, then the two triangles are similar.

Knowledge BOOSTER

1. All congruent triangles are similar but the similar triangles need not be congruent.
2. **Mid-point Theorem:** The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it.

3. **Angle Bisector Theorem:** The internal bisector of an angle of a triangle divides the opposite side in two segments that are proportional to the other two sides of the triangle.



In $\triangle ABC$, $\frac{AB}{AC} = \frac{BD}{DC}$

4. If two triangles are similar, then their corresponding sides, medians and altitudes are proportional.

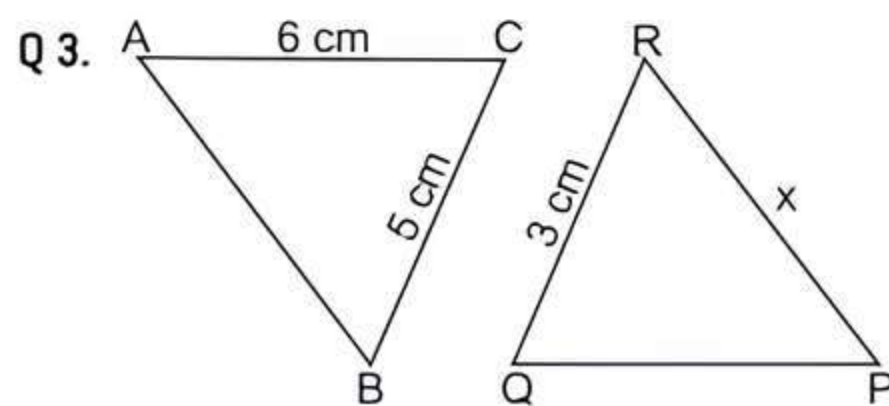


Practice Exercise



Multiple Choice Questions

- Q 1. Two polygons have same number of sides are similar, if:
 - a. their corresponding sides are proportional
 - b. their corresponding angles are equal
 - c. Both a. and b.
 - d. None of the above
- Q 2. If $\triangle ABC \sim \triangle PQR$ with $\angle A = 32^\circ$ and $\angle R = 65^\circ$, then the measures of $\angle B$ is: [CBSE 2023]
 - a. 32°
 - b. 65°
 - c. 83°
 - d. 97°



In the given figure, $\triangle ABC \sim \triangle QPR$. If $AC = 6$ cm, $BC = 5$ cm, $QR = 3$ cm and $PR = x$, then the value of x is:

- [CBSE 2023]
- a. 3.6 cm
 - b. 2.5 cm
 - c. 10 cm
 - d. 3.2 cm

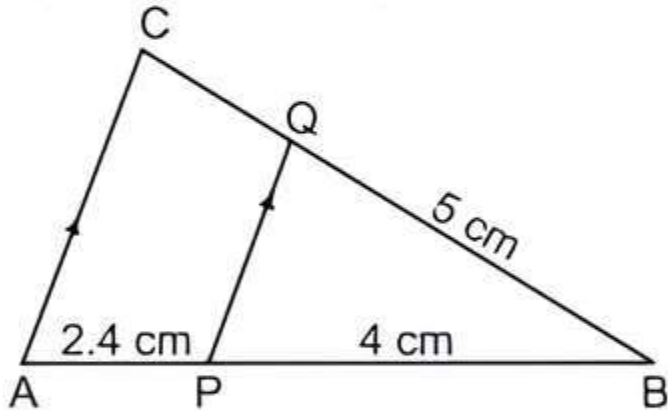
Q 4. In a $\triangle ABC$, it is given that $AB = 6$ cm, $AC = 8$ cm and AD is the bisector of $\angle A$. Then, $BD : DC =$

- a. 3 : 4 b. 9 : 16 c. 4 : 3 d. $\sqrt{3} : 2$

Q 5. $\triangle ABC \sim \triangle PQR$. If AM and PN are altitudes of $\triangle ABC$ and $\triangle PQR$ respectively and $AB^2 : PQ^2 = 4 : 9$, then $AM : PN =$ [CBSE SQP 2021 Term-I]

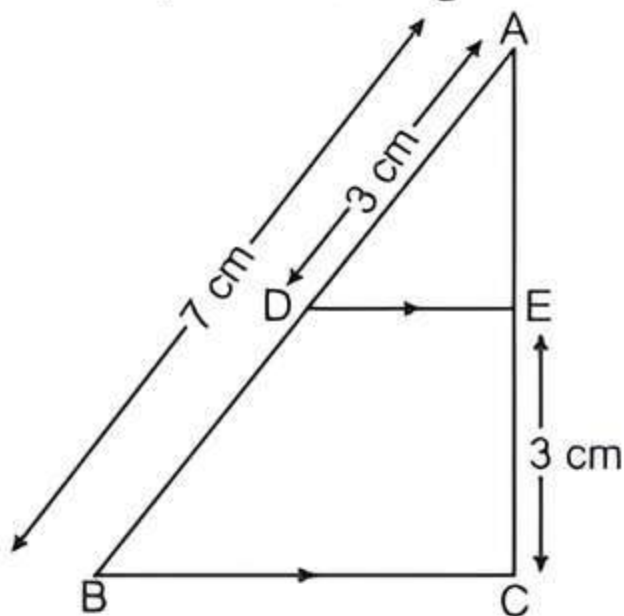
- a. 16 : 81 b. 4 : 9 c. 3 : 2 d. 2 : 3

Q 6. In the given figure, $PQ \parallel AC$. If $BP = 4$ cm, $AP = 2.4$ cm and $BQ = 5$ cm, then length of BC is: [CBSE 2023]



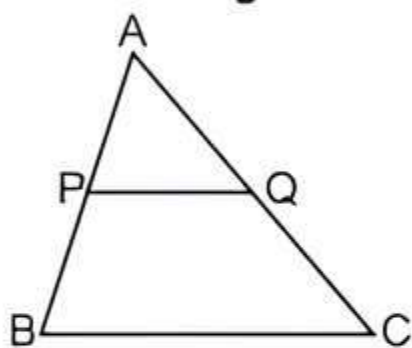
- a. 8 cm b. 3 cm
c. 0.3 cm d. $\frac{25}{3}$ cm

Q 7. In the given figure, $DE \parallel BC$. If $AD = 3$ cm, $AB = 7$ cm and $EC = 3$ cm, then the length of AE is: [CBSE 2023]



- a. 2 cm b. 2.25 cm
c. 3.5 cm d. 4 cm

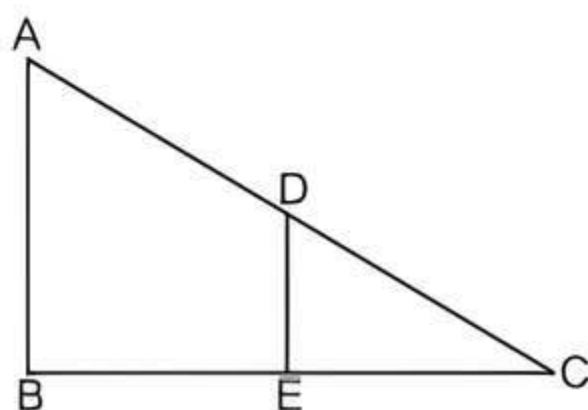
Q 8. In $\triangle ABC$, $PQ \parallel BC$. If $PB = 6$ cm, $AP = 4$ cm, $AQ = 8$ cm, find the length of AC . [CBSE 2023]



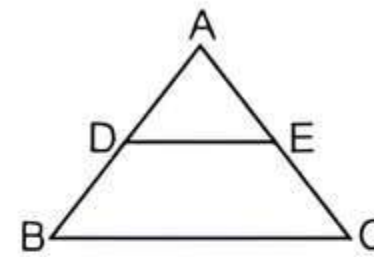
- a. 12 cm b. 20 cm
c. 6 cm d. 14 cm

Q 9. In $\triangle ABC$, $DE \parallel AB$. If $AB = a$, $DE = x$, $BE = b$ and $EC = c$. Express x in terms of a , b and c . [CBSE SQP 2023-24]

- a. $\frac{ac}{b}$
b. $\frac{ac}{b+c}$
c. $\frac{ab}{c}$
d. $\frac{ab}{b+c}$



Q 10. In the given figure, if $DE \parallel BC$, $AD = 3$ cm, $BD = 4$ cm and $BC = 14$ cm, then DE equals: [CBSE SQP 2021 Term-I]



- a. 7 cm b. 6 cm c. 4 cm d. 3 cm

Q 11. In $\triangle ABC$ and $\triangle DEF$, $\angle B = \angle E$, $\angle F = \angle C$ and $AB = \frac{1}{2}DE$. Then, the two triangles are: [CBSE 2021 Term-I]

- a. congruent but not similar
b. similar but not congruent
c. neither congruent nor similar
d. congruent as well as similar

Q 12. It is given that $\triangle ABC \sim \triangle DFE$, $\angle A = 30^\circ$, $\angle C = 50^\circ$, $AB = 5$ cm, $AC = 8$ cm and $DF = 7.5$ cm. Then, which of the following is true? [NCERT EXEMPLAR]

- a. $DE = 12$ cm, $\angle F = 50^\circ$ b. $DE = 12$ cm, $\angle F = 100^\circ$
c. $EF = 12$ cm, $\angle D = 100^\circ$ d. $EF = 12$ cm, $\angle D = 30^\circ$

Q 13. If the corresponding medians of two similar triangles are in the ratio 5 : 7, then the ratio of their corresponding sides is: [CBSE 2015]

- a. 25 : 49 b. 5 : 7
c. 7 : 5 d. 49 : 25

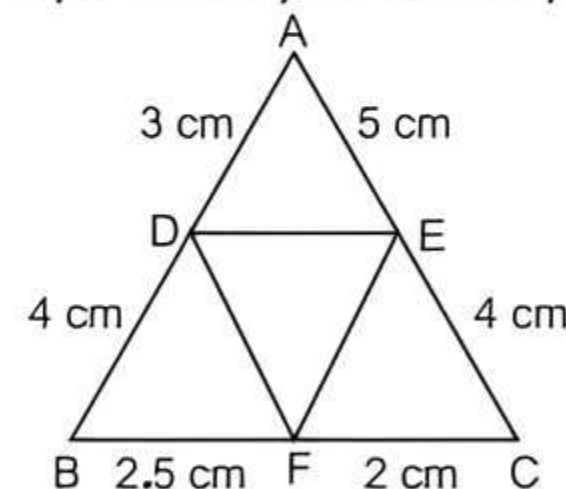
Q 14. The parallel sides of a trapezium are 3 cm and 9 cm. The non-parallel sides are 4 cm and 6 cm. A line parallel to the base divides the trapezium into two trapeziums of equal perimeters. The ratio in which each of the non-parallel sides is divided, is:

- a. 4 : 5 b. 3 : 2 c. 4 : 1 d. 3 : 1

Q 15. ABCD is a trapezium with $AD \parallel BC$ and $AD = 4$ cm. If the diagonals AC and BD intersect each other at O such that $AO/OC = DO/OB = 1/2$, then $BC =$ [CBSE SQP 2022-23]

- a. 6 cm b. 7 cm c. 8 cm d. 9 cm

Q 16. In the given figure, $AD = 3$ cm, $AE = 5$ cm, $BD = 4$ cm, $CE = 4$ cm, $CF = 2$ cm, $BF = 2.5$ cm, then:

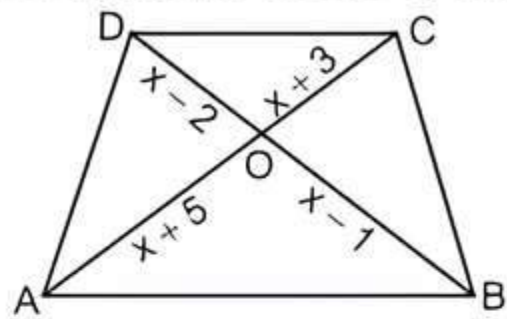


- a. $DE \parallel BC$ b. $DF \parallel AC$
c. $EF \parallel AB$ d. None of these

Q 17. It is given that, $\triangle ABC \sim \triangle EDF$, such that $AB = 5$ cm, $AC = 7$ cm, $DF = 15$ cm and $DE = 12$ cm, then the sum of the remaining sides of the triangles is:

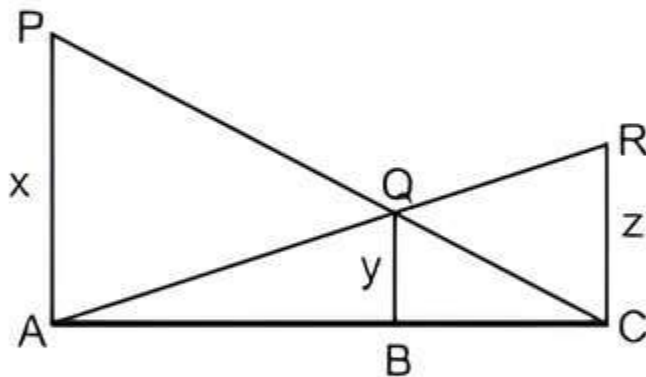
- a. 23.05 cm b. 16.8 cm
c. 6.25 cm d. 24 cm

Q 18. In the given figure, if $AB \parallel DC$, find the value of x .



- a. 5 b. 7 c. 6 d. 4

Q 19. In the given figure, PA , QB and RC are each perpendicular to AC . If $x = 8$ cm and $z = 6$ cm, then y is equal to: [CBSE 2021 Term-I]

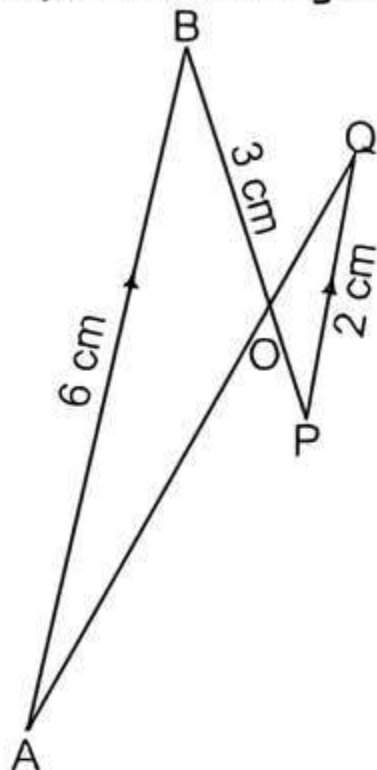


- a. $\frac{56}{7}$ cm b. $\frac{7}{56}$ cm
c. $\frac{25}{7}$ cm d. $\frac{24}{7}$ cm

Q 20. If $\Delta PQR \sim \Delta ABC$; $PQ = 6$ cm, $AB = 8$ cm and the perimeter of ΔABC is 36 cm, then the perimeter of ΔPQR is: [CBSE 2023]

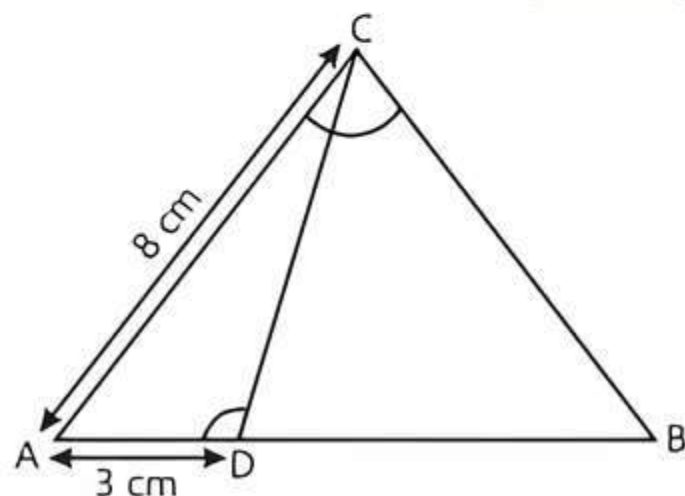
- a. 20.25 cm b. 27 cm
c. 48 cm d. 64 cm

Q 21. In the given figure, $AB \parallel PQ$. If $AB = 6$ cm, $PQ = 2$ cm and $OB = 3$ cm, then the length of OP is: [CBSE 2023]



- a. 9 cm b. 3 cm
c. 4 cm d. 1 cm

Q 22. In the given figure, if $\angle ACB = \angle CDA$, $AC = 8$ cm, $AD = 3$ cm, then BD is: [CBSE SQP 2021 Term-I]

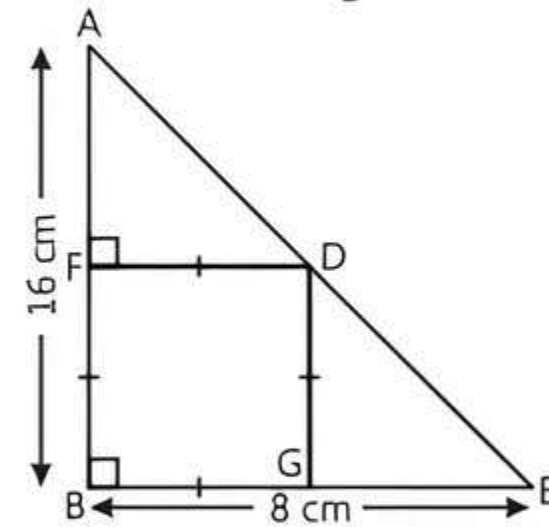


- a. $\frac{22}{3}$ cm b. $\frac{26}{3}$ cm
c. $\frac{55}{3}$ cm d. $\frac{64}{3}$ cm

Q 23. A vertical stick 12 m long casts a shadow 8 m long on the ground. At the same time, a tower casts the shadow 40 m long on the ground. Then, the height of the tower is:

- a. 65 m b. 60 m c. 70 m d. 72 m

Q 24. Sides AB and BE of a right triangle, right angled at B are of lengths 16 cm and 8 cm respectively. The length of the side of largest square $FDGB$ that can be inscribed in the triangle ABE is:



- a. $\frac{32}{3}$ cm b. $\frac{16}{3}$ cm
c. $\frac{8}{3}$ cm d. $\frac{4}{3}$ cm



Assertion & Reason Type Questions

Directions (Q.Nos. 25-29): In the following questions given below, there are two statements marked as Assertion (A) and Reason (R). Read the statements and choose the correct option:

- a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true, but Reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true

Q 25. Assertion (A): All regular polygons of the same number of sides such as equilateral triangles, squares etc, are similar.

Reason (R): Two polygons of the same number of sides are said to be similar, if their corresponding angles are equal and lengths of corresponding sides are proportional.

Q 26. Assertion (A): In a ΔABC , if $DE \parallel BC$ and intersects AB at D and AC at E , then $\frac{AB}{AD} = \frac{AC}{AE}$.

Reason (R): If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then these sides are divided in the same ratio.

Q 27. Assertion (A): If the bisector of an angle of a triangle bisects the opposite side, then the triangle is isosceles.

Reason (R): The internal bisector of an angle of a triangle divides the opposite side internally in the ratio of the sides containing the angle.

Q 28. Assertion (A): In a $\triangle ABC$, D and E are points on sides AB and AC respectively such that $BD = CE$. If $\angle B = \angle C$, then DE is not parallel to BC.
Reason (R): If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Q 29. Assertion (A): In $\triangle ABC$, $DE \parallel BC$, such that $AD = (7x - 4)$ cm, $AE = (5x - 2)$ cm, $DB = (3x + 4)$ cm and $EC = 3x$ cm then x is equal to 5.
Reason (R): If a line is drawn parallel to one side of triangle to intersect the other two sides at a distinct point, then the other two sides are divided in the same ratio.

Fill in the Blanks Type Questions

- Q 30. Two triangles are similar, if their corresponding sides are [NCERT EXERCISE; CBSE 2020]
- Q 31. In $\triangle ABC$ and $\triangle DEF$, if $\angle B = \angle E$, $\angle F = \angle C$ and $AB = 2DE$. Then, the two triangles are

- Q 32. If in two triangles DEF and PQR, $\angle D = \angle Q$ and $\angle R = \angle E$, then $\frac{DE}{QR} = \frac{DF}{PQ} = \dots\dots\dots$
- Q 33. The line segment joining the mid-points of any two sides of a triangle is to the third side.
- Q 34. All congruent triangles are similar but the similar triangles need not to be

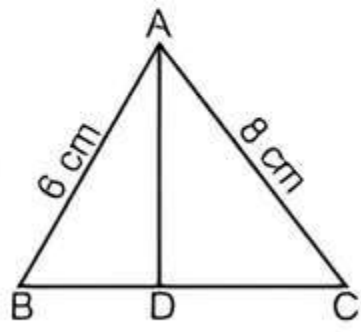
True/False Type Questions

- Q 35. If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio. [NCERT EXEMPLAR]
- Q 36. If AD and PM are medians of $\triangle ABC$ and $\triangle PQR$ respectively, where $\triangle ABC \sim \triangle PQR$, then $\frac{AB}{PQ} = \frac{AD}{PM}$. [NCERT EXERCISE]
- Q 37. In two triangles, if one pair of corresponding sides are proportional and the included angles are also equal, then two triangles are similar.

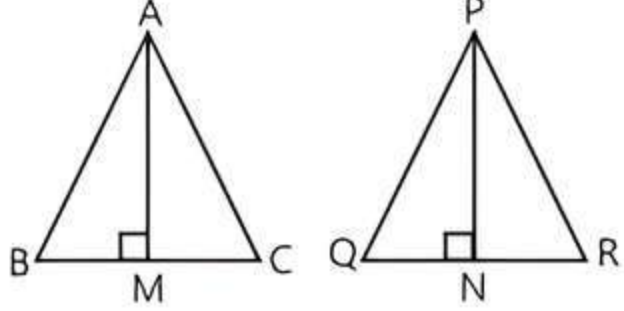
Solutions



1. (c) Two polygons of the same number of sides are similar if their corresponding angles are equal and their corresponding sides are in the same ratio i.e. proportional.
2. (c) Given $\triangle ABC \sim \triangle PQR$
 $\Rightarrow \angle A = \angle P, \angle B = \angle Q$ and $\angle C = \angle R$
 $\therefore \angle A = \angle P = 32^\circ$ and $\angle R = \angle C = 65^\circ$
 Now, in $\triangle ABC$,
 $\angle A + \angle B + \angle C = 180^\circ$
 $\Rightarrow 32^\circ + \angle B + 65^\circ = 180^\circ$
 $\Rightarrow \angle B = 180^\circ - 97^\circ = 83^\circ$
3. (b) Given. $\triangle ABC \sim \triangle QPR$
 $\therefore \frac{AB}{QP} = \frac{BC}{PR} = \frac{CA}{RQ}$
 $\Rightarrow \frac{BC}{PR} = \frac{CA}{RQ} \Rightarrow \frac{5}{x} = \frac{6}{3}$
 $\Rightarrow x = \frac{5 \times 3}{6} = \frac{5}{2} = 2.5$ cm

4. (a) We know that, the bisector of an angle of a triangle divides the opposite side in the ratio of the sides containing the angle.
 $\therefore BD : DC = AB : AC = 6 : 8 = 3 : 4$



5. (d) Given, $\triangle ABC \sim \triangle PQR$



 **TIP**  If two triangles are similar, then their corresponding sides and altitudes are in same proportion.

AM and PN are altitudes of $\triangle ABC$ and $\triangle PQR$ respectively.

Also, $\frac{AB^2}{PQ^2} = \frac{4}{9} \Rightarrow \frac{AB}{PQ} = \frac{2}{3}$... (1)

As we know that, Ratio of altitudes = Ratio of sides for similar triangles.

So, $\frac{AM}{PN} = \frac{AB}{PQ} = \frac{2}{3}$

6. (a) In $\triangle ABC$, $PQ \parallel AC$
 $\therefore \frac{AP}{BP} = \frac{CQ}{BQ}$ [By Thales theorem]
 $\Rightarrow \frac{2.4}{4} = \frac{CQ}{5}$
 $\Rightarrow CQ = \frac{2.4 \times 5}{4} = 3$ cm
 \therefore Length of BC = BQ + CQ = 5 + 3 = 8 cm

7. (b) In $\triangle ABC$, $BC \parallel DE$
 $\therefore \frac{AD}{BD} = \frac{AE}{CE}$ [By Thales theorem]
 $\Rightarrow \frac{AD}{BD} + 1 = \frac{AE}{CE} + 1$
 $\Rightarrow \frac{AD + BD}{BD} = \frac{AE + CE}{CE}$

$$\Rightarrow \frac{AB}{AB-AD} = \frac{AE+CE}{CE}$$

$$\Rightarrow \frac{7}{7-3} = \frac{AE+3}{3}$$

$$\Rightarrow \frac{7}{4} = \frac{AE}{3} + 1$$

$$\Rightarrow \frac{AE}{3} = \frac{7}{4} - 1 = \frac{3}{4}$$

$$\Rightarrow AE = \frac{9}{4} = 2.25 \text{ cm}$$

8. (b) Given that.
In $\triangle ABC$, $PQ \parallel BC$
 $PB = 6 \text{ cm}$, $AP = 4 \text{ cm}$, $AQ = 8 \text{ cm}$

$$\therefore \frac{AP}{AB} = \frac{AQ}{AC}$$

$$\Rightarrow \frac{AP}{AP+PB} = \frac{AQ}{AC} \quad (\text{By BPT})$$

$$\Rightarrow \frac{4}{4+6} = \frac{8}{AC} \Rightarrow \frac{4}{10} = \frac{8}{AC}$$

$$\Rightarrow AC = \frac{8 \times 10}{4} = 20 \text{ cm}$$

9. (b) Given $AB = a$, $DE = x$
 $BE = b$ and $EC = c$

In $\triangle ABC$, $DE \parallel AB$
By basic proportionality theorem,

$$\Rightarrow \frac{DE}{ABC} = \frac{EC}{BC}$$

$$\Rightarrow \frac{DE}{AB} = \frac{EC}{BE+EC}$$

$$\Rightarrow \frac{x}{a} = \frac{c}{b+c} \Rightarrow x = \frac{ac}{b+c}$$

10. (b) Given, $AD = 3 \text{ cm}$, $BD = 4 \text{ cm}$, $BC = 14 \text{ cm}$
and $DE \parallel BC$

$$\therefore \triangle ADE \sim \triangle ABC \quad (\text{By AA similarity})$$

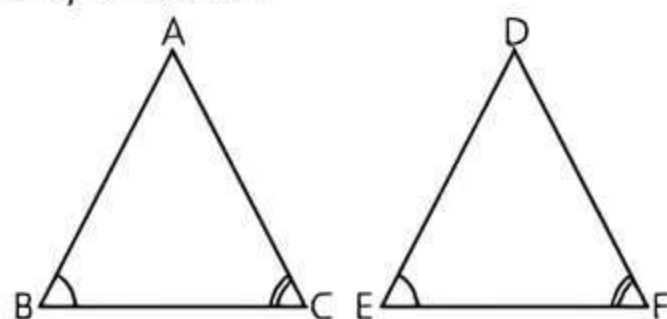
$$\Rightarrow \frac{AD}{AB} = \frac{DE}{BC} \Rightarrow \frac{AD}{AD+BD} = \frac{DE}{BC}$$

$$\Rightarrow \frac{3}{3+4} = \frac{DE}{14} \Rightarrow DE = \frac{3 \times 14}{7} = 6 \text{ cm}$$

11. (b) In $\triangle ABC$ and $\triangle DEF$,

$$\angle B = \angle E, \angle C = \angle F \text{ and } AB = \frac{1}{2}DE.$$

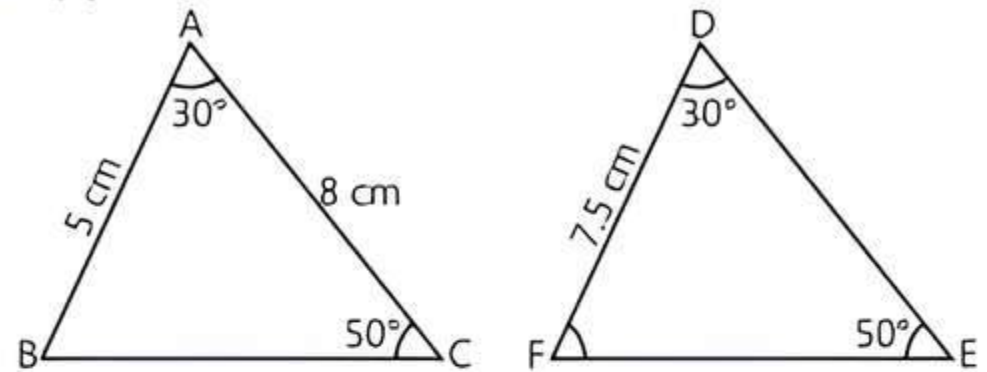
We know that, if in two triangles corresponding two angles are same, then they are similar by AA similarity criterion.



Since, $AB \neq DE$

Therefore, $\triangle ABC$ and $\triangle DEF$ are not congruent.

12. (b) Given, $\triangle ABC \sim \triangle DFE$



Then $\angle A = \angle D = 30^\circ$,
 $\angle C = \angle E = 50^\circ$
 $\therefore \angle B = \angle F = 180^\circ - (30^\circ + 50^\circ) = 100^\circ$
Also, $AB = 5 \text{ cm}$, $AC = 8 \text{ cm}$ and $DF = 7.5 \text{ cm}$

$$\therefore \frac{AB}{DF} = \frac{AC}{DE} \Rightarrow \frac{5}{7.5} = \frac{8}{DE}$$

$$\Rightarrow DE = \frac{8 \times 7.5}{5} = 12 \text{ cm}$$

Hence, $DE = 12 \text{ cm}$, $\angle F = 100^\circ$

13. (b) Let ABC and PQR be two similar triangles with medians AD and PS respectively.

Then, $\frac{AD}{PS} = \frac{5}{7}$ (Given)

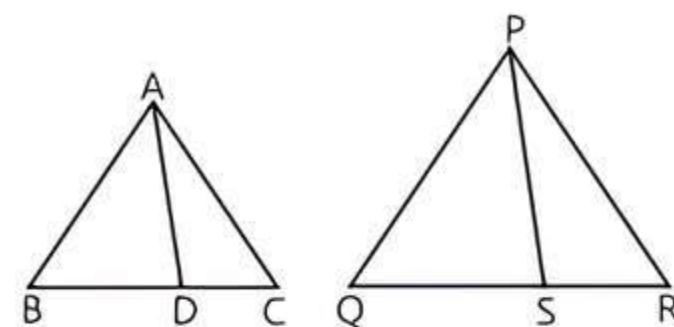
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The ratios of the medians of two similar triangles is equal to the ratio of their corresponding sides.

$$\therefore \triangle ABC \sim \triangle PQR$$

$$\frac{AB}{PQ} = \frac{AD}{PS}$$

$$\Rightarrow \frac{AB}{PQ} = \frac{5}{7}$$

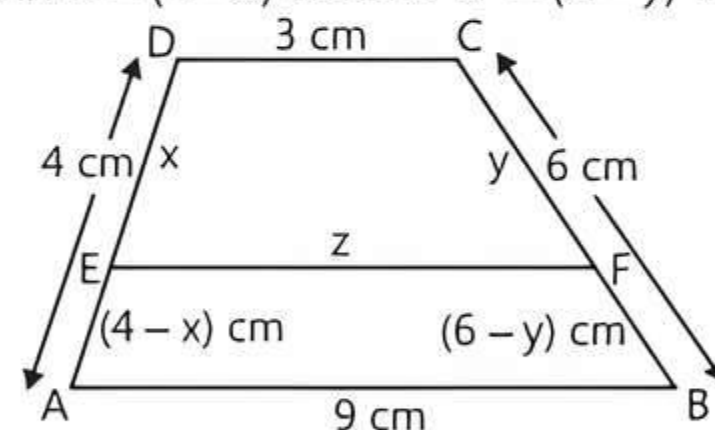


Hence, the ratio of corresponding sides is $5 : 7$.

14. (c) Given, $AD = 4 \text{ cm}$ and $BC = 6 \text{ cm}$.

Let $DE = x \text{ cm}$, $CF = y \text{ cm}$ and $EF = z \text{ cm}$

Then, $AE = (4 - x) \text{ cm}$ and $BF = (6 - y) \text{ cm}$



According to the given condition,

Perimeter of $DCFE =$ Perimeter of $ABFE$

$$\Rightarrow x + 3 + y + z = 9 + 6 - y + z + 4 - x$$

$$\Rightarrow x + 3 + y = 19 - x - y$$

$$\Rightarrow 2x + 2y = 16$$

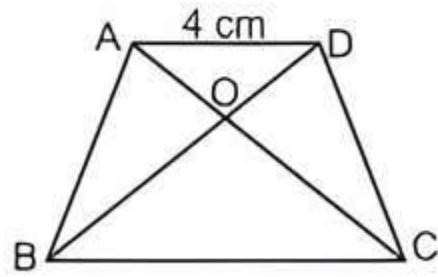
$$\Rightarrow x + y = 8$$

$$\Rightarrow y = 8 - x$$

We know that line parallel to the base divides the two non-parallel lines in the same ratio.

$$\begin{aligned} \therefore \frac{DE}{AE} &= \frac{CF}{BF} \\ \Rightarrow \frac{x}{4-x} &= \frac{y}{6-y} \\ \Rightarrow \frac{x}{4-x} &= \frac{8-x}{6-8+x} \\ \Rightarrow x(x-2) &= (8-x)(4-x) \\ \Rightarrow x^2 - 2x &= 32 - 12x + x^2 \\ \Rightarrow 10x &= 32 \text{ cm} \\ \Rightarrow x &= 3.2 \text{ cm} \\ \therefore \text{Ratio} &= \frac{DE}{AE} = \frac{3.2}{4-3.2} = \frac{3.2}{0.8} = \frac{4}{1} = 4:1 \end{aligned}$$

15. (c) Given. $AD = 4 \text{ cm}$
and $\frac{AO}{OC} = \frac{DO}{OB} = \frac{1}{2}$... (1)

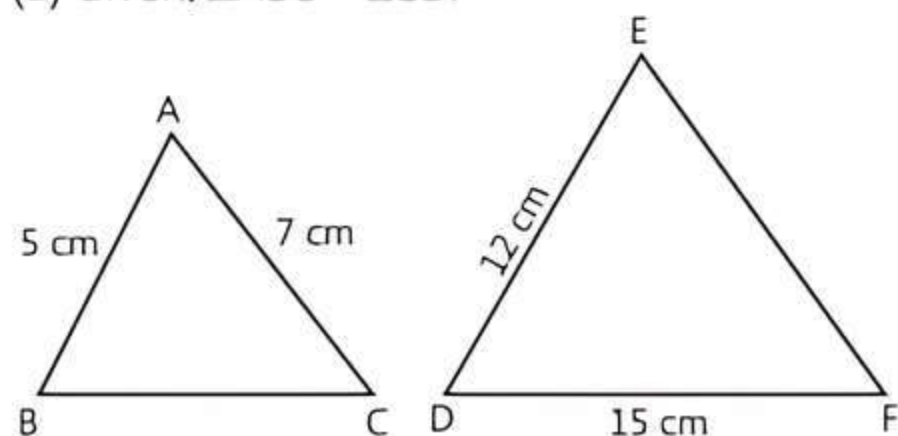


In $\triangle AOD$ and $\triangle COB$
 $\angle ADO = \angle CBO$
 ($\because AD \parallel BC$, so alternate angles are equal)
 $\angle AOD = \angle COB$ (Vertically opposite angles)
 and $\angle OAD = \angle OCB$
 (Alternate angles are equal)
 $\therefore \triangle AOD \sim \triangle COB$ (By AAA similarity rule)
 $\Rightarrow \frac{AO}{OC} = \frac{OD}{OB} = \frac{AD}{BC}$
 $\Rightarrow \frac{1}{2} = \frac{4}{BC}$ [From eq. (1)]
 $\Rightarrow BC = 8 \text{ cm}$

16. (c) Given. $AD = 3 \text{ cm}$, $AE = 5 \text{ cm}$, $BD = 4 \text{ cm}$,
 $CE = 4 \text{ cm}$, $CF = 2 \text{ cm}$, $BF = 2.5 \text{ cm}$

Here $\frac{CF}{FB} = \frac{2}{2.5}$
 $= \frac{20}{25} = \frac{4}{5}$
 and $\frac{CE}{AE} = \frac{4}{5}$
 $\therefore \frac{CF}{FB} = \frac{CE}{AE}$
 $\Rightarrow EF \parallel AB$ (By converse of Thales theorem)

17. (a) Given. $\triangle ABC \sim \triangle EDF$



Since, $\triangle ABC \sim \triangle EDF$

$$\begin{aligned} \therefore \frac{AB}{ED} &= \frac{BC}{DF} = \frac{AC}{EF} \\ \Rightarrow \frac{5}{12} &= \frac{7}{EF} = \frac{BC}{15} \end{aligned}$$

On taking first and second ratios, we get

$$\frac{5}{12} = \frac{7}{EF} \Rightarrow EF = \frac{7 \times 12}{5} = 16.8 \text{ cm}$$

On taking first and third ratios, we get

$$\frac{5}{12} = \frac{BC}{15} \Rightarrow BC = \frac{5 \times 15}{12} = 6.25 \text{ cm}$$

Now, sum of the remaining sides of triangle.
 $= EF + BC = 16.8 + 6.25 = 23.05 \text{ cm}$

18. (b) Given. $AB \parallel DC$
 $\therefore \angle ODC = \angle OBA$ (Alternate interior angles)
 and $\angle OCD = \angle OAB$ (Alternate interior angles)
 $\therefore \triangle DOC \sim \triangle BOA$ (By AA similarity criterion)

Thus, $\frac{OD}{OB} = \frac{OC}{OA}$

$$\begin{aligned} \Rightarrow \frac{x-2}{x-1} &= \frac{x+3}{x+5} \\ \Rightarrow (x-2)(x+5) &= (x+3)(x-1) \\ \Rightarrow x^2 + 3x - 10 &= x^2 + 2x - 3 \\ \Rightarrow x &= 7 \end{aligned}$$

19. (d) $\triangle PAC$ and $\triangle QBC$ is similar.

$$\therefore \frac{AP}{BQ} = \frac{AC}{BC} \Rightarrow \frac{x}{y} = \frac{AC}{BC}$$

or $\frac{y}{x} = \frac{BC}{AC}$... (1)

Also, $\triangle RAC$ and $\triangle QAB$ is similar.

$$\therefore \frac{CR}{BQ} = \frac{AC}{AB} \Rightarrow \frac{z}{y} = \frac{AC}{AB}$$

or $\frac{y}{z} = \frac{AB}{AC}$... (2)

Adding eqs. (1) and (2), we get

$$\begin{aligned} \frac{y}{x} + \frac{y}{z} &= \frac{BC}{AC} + \frac{AB}{AC} \\ \Rightarrow y \left(\frac{1}{x} + \frac{1}{z} \right) &= \frac{BC+AB}{AC} \\ \Rightarrow y \left(\frac{1}{x} + \frac{1}{z} \right) &= \frac{AC}{AC} \\ \Rightarrow y \left(\frac{1}{8} + \frac{1}{6} \right) &= 1 \Rightarrow y \left(\frac{6+8}{48} \right) = 1 \\ \Rightarrow y &= \frac{48}{14} = \frac{24}{7} \text{ cm} \end{aligned}$$

20. (b) Given. $\triangle PQR \sim \triangle ABC$

$$\therefore \frac{PQ}{AB} = \frac{QR}{BC} = \frac{PR}{AC} = \frac{PQ+QR+PR}{AB+BC+AC}$$

$$\Rightarrow \frac{PQ}{AB} = \frac{\text{perimeter of } \triangle PQR}{\text{perimeter of } \triangle ABC} \dots (1)$$

Given $PQ = 6$ cm, $AB = 8$ cm and perimeter of $\triangle ABC = 36$ cm

From eq. (1), we get

$$\frac{6}{8} = \frac{\text{Perimeter of } \triangle PQR}{36}$$

$$\text{Perimeter of } \triangle PQR = \frac{6 \times 36}{8} = 27 \text{ cm}$$

21. (d) In the given figure, $AB \parallel PQ$

In $\triangle PQR$ and $\triangle BOA$,

$$\angle OPQ = \angle OBA \quad (\text{Alternate interior angles})$$

$$\angle OQP = \angle OAB \quad (\text{Alternative interior angles})$$

and $\angle POQ = \angle BOA$ (Vertically opposite angles)

$$\therefore \triangle POQ \sim \triangle BOA$$

$$\Rightarrow \frac{PO}{BO} = \frac{OQ}{OA} = \frac{PQ}{BA}$$

Given $AB = 6$ cm, $PQ = 2$ cm and $OB = 3$ cm

$$\Rightarrow \frac{PO}{BO} = \frac{PQ}{BA} \Rightarrow \frac{OP}{3} = \frac{2}{6}$$

$$\Rightarrow OP = \frac{2 \times 3}{6} = 1 \text{ cm}$$

22. (c) Given, $AC = 8$ cm, $AD = 3$ cm

In $\triangle ACD$ and $\triangle ABC$,

$$\angle CDA = \angle ACB \quad (\text{Given})$$

$$\angle CAD = \angle CAB \quad (\text{Common})$$

$$\therefore \triangle ACD \sim \triangle ABC \quad (\text{By AA similarity})$$

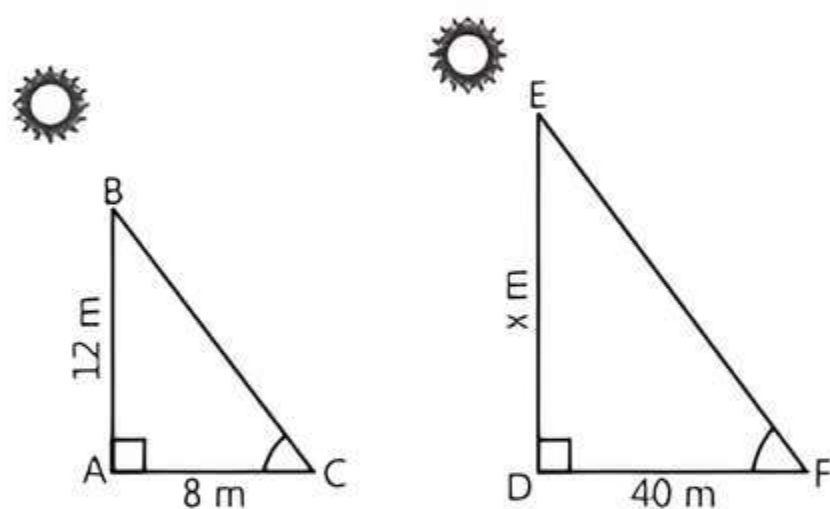
$$\Rightarrow \frac{AC}{AB} = \frac{AD}{AC} \quad (\text{By CPCT})$$

$$\Rightarrow \frac{8}{AB} = \frac{3}{8}$$

$$\Rightarrow AB = \frac{64}{3} \text{ cm}$$

So, $BD = AB - AD = \frac{64}{3} - 3 = \frac{55}{3} \text{ cm}$

23. (b) Let AB be the vertical stick and AC be its shadow. Also, let DE be the vertical tower and DF be its shadow. Join BC and EF . Let $DE = x$ m



We have, $AB = 12$ m, $AC = 8$ m and $DF = 40$ m

In $\triangle ABC$ and $\triangle DEF$, we have

$$\angle A = \angle D = 90^\circ$$

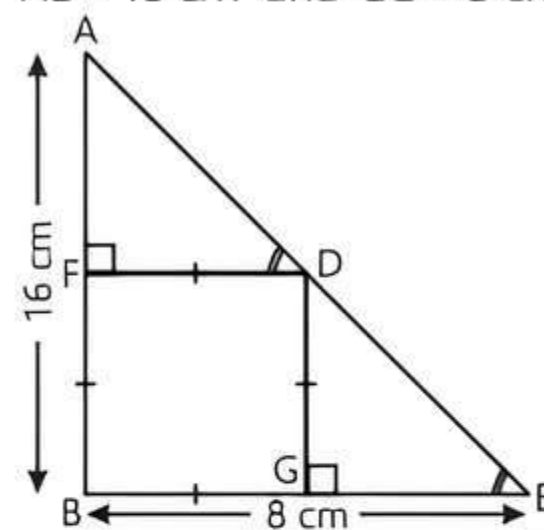
$$\angle C = \angle F \quad (\text{Angular elevation of the sun})$$

$$\therefore \triangle ABC \sim \triangle DEF \quad (\text{By AA similarity criterion})$$

$$\Rightarrow \frac{AB}{DE} = \frac{AC}{DF} \Rightarrow \frac{12}{x} = \frac{8}{40}$$

$$\Rightarrow \frac{12}{x} = \frac{1}{5} \Rightarrow x = 60 \text{ m}$$

24 (b) Given, $\triangle ABE$ is a right-angled triangle with $AB = 16$ cm and $BE = 8$ cm



Let each side of square $FDGB$ be x cm.

In $\triangle AFD$ and $\triangle DGE$,

$$\angle AFD = \angle DGE \quad (\text{Each } 90^\circ)$$

$$\angle ADF = \angle DEG \quad (\text{From figure})$$

$$\therefore \triangle AFD \sim \triangle DGE \quad (\text{By AA similarity})$$

$$\Rightarrow \frac{AF}{DG} = \frac{FD}{GE}$$

$$\Rightarrow \frac{AB - FB}{DG} = \frac{FD}{BE - BG} \quad (\text{From figure})$$

$$\Rightarrow \frac{16 - x}{x} = \frac{x}{8 - x}$$

$$\Rightarrow 128 - 24x + x^2 = x^2$$

$$\Rightarrow 24x = 128$$

$$\Rightarrow x = \frac{128}{24} = \frac{16}{3} \text{ cm}$$

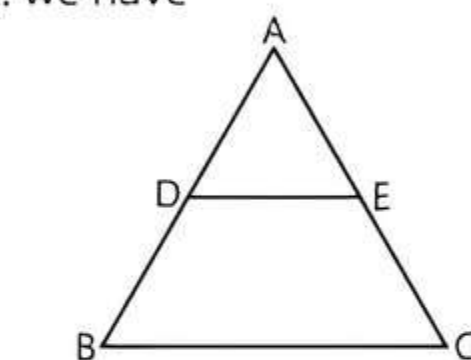
25. (a) **Assertion (A)**: Two polygons of the same number of sides are similar, if their corresponding angles are equal and corresponding sides are proportional.

\therefore In equilateral triangles or squares, each angle is equal and sides are also proportional, therefore all regular polygons are similar.

Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

26. (a) **Assertion (A)**: In $\triangle ABC$, $DE \parallel BC$, by using Thale's theorem, we have



$$\frac{AD}{DB} = \frac{AE}{EC} \Rightarrow \frac{DB}{AD} = \frac{EC}{AE}$$

$$\Rightarrow 1 + \frac{DB}{AD} = 1 + \frac{EC}{AE}$$

$$\Rightarrow \frac{AD + DB}{AD} = \frac{AE + EC}{AE}$$

$$\Rightarrow \frac{AB}{AD} = \frac{AC}{AE}$$

So, Assertion (A) is true.

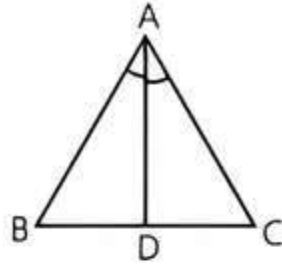
Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

27. (a) **Assertion (A):** In $\triangle ABC$, AD is the bisector of $\angle A$.

$$\therefore \frac{AB}{AC} = \frac{BD}{DC}$$

$$\Rightarrow \frac{AB}{AC} = 1$$



(\because D is the mid-point of BC, \therefore $BD = DC$)

$$\Rightarrow AB = AC$$

Hence, $\triangle ABC$ is an isosceles.

So, Assertion (A) is true.

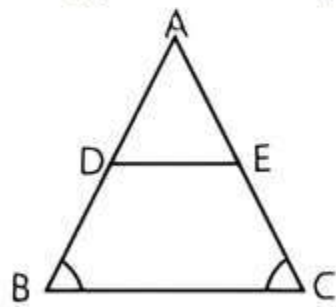
Reason (R): It is also true.

Hence, both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

28. (d) **Assertion (A):** In $\triangle ABC$, we have $\angle B = \angle C$

$$\Rightarrow AC = AB$$

(\because Sides opposite to equal angles are equal)



$$\Rightarrow AE + EC = AD + DB$$

$$\Rightarrow AE + CE = AD + DB$$

$$\Rightarrow AE + CE = AD + CE \quad [BD = CE \text{ (Given)}]$$

$$\Rightarrow AE = AD$$

Thus, we have

$$AD = AE$$

$$\text{and } BD = CE$$

$$\therefore \frac{AD}{BD} = \frac{AE}{CE} \Rightarrow \frac{AD}{DB} = \frac{AE}{EC}$$

$$\Rightarrow DE \parallel BC$$

So, Assertion (A) is false.

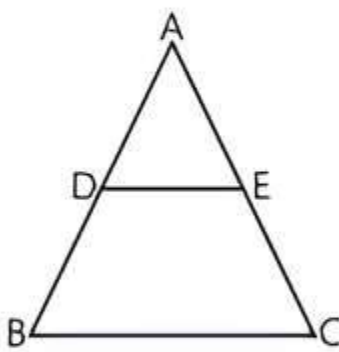
Reason (R): It is true statement.

Hence, Assertion (A) is false but Reason (R) is true.

29. (d) **Assertion (A):** We have,

$$\frac{AD}{DB} = \frac{AE}{EC} \quad [\because DE \parallel BC]$$

$$\Rightarrow \frac{7x-4}{3x+4} = \frac{5x-2}{3x}$$



$$\Rightarrow 21x^2 - 12x = 15x^2 + 20x - 6x - 8$$

$$\Rightarrow 6x^2 - 26x + 8 = 0$$

$$\Rightarrow 3x^2 - 13x + 4 = 0$$

TRICK

Product of extreme terms = $3 \times 4 = 12$

$$\therefore 12 = 6 \times 2 = 3 \times 4 = 12 \times 1$$

Here, we will take 12 and 1 as a factors of 12. So, middle term

$$-13 = -12 - 1$$

$$\Rightarrow 3x^2 - 12x - x + 4 = 0$$

$$\Rightarrow 3x(x-4) - 1(x-4) = 0$$

$$\Rightarrow (x-4)(3x-1) = 0$$

$$\Rightarrow x = 4, \frac{1}{3}$$

So, Assertion (A) is false.

Reason (R): It is true statement.

Hence, Assertion (A) is false but Reason (R) is true.

30. Proportional

31. In $\triangle ABC$ and $\triangle DEF$,

$$\angle B = \angle E \quad \text{(Given)}$$

$$\angle C = \angle F \quad \text{(Given)}$$

$$\therefore \triangle ABC \sim \triangle DEF \quad \text{(By AA similarity)}$$

32. In $\triangle DEF$ and $\triangle QRP$,

$$\angle D = \angle Q \quad \text{(Given)}$$

$$\angle E = \angle R \quad \text{(Given)}$$

$$\therefore \triangle DEF \sim \triangle QRP \quad \text{(By AA similarity)}$$

$$\Rightarrow \frac{DE}{QR} = \frac{DF}{QP} = \frac{EF}{RP}$$

33. Parallel

34. Congruent

35. True

36. True

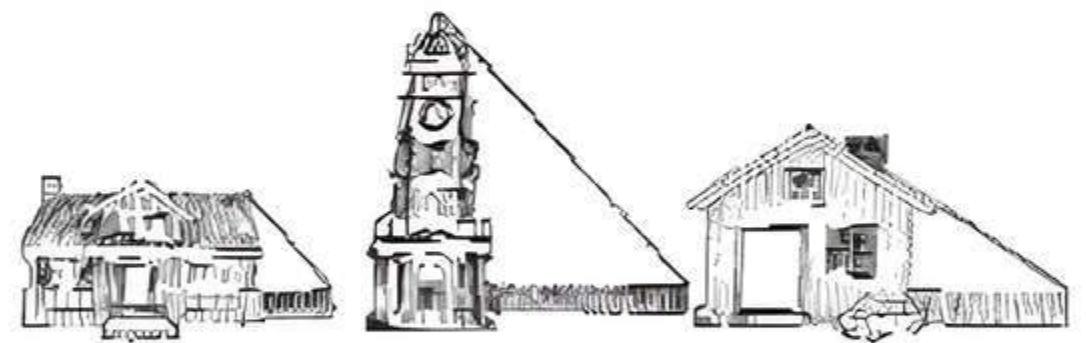
37. True

Case Study Based Questions

Case Study 1

Digvijay is trying to find the average height of a tower near his house. He is using the properties of similar triangles. The height of Digvijay's house is 20 m when Digvijay's house casts a shadow 10 m long on the ground.

At the same time, the tower casts a shadow 50 m long on the ground and the house of Anshul casts 20 m shadow on the ground.



Based on the above information, solve the following questions:

Q1. The height of the tower is:

- a. 10 m b. 20 m c. 50 m d. 100 m

Q2. When Digvijay's house casts a shadow of 18 cm, the length of the shadow of the tower is:

- a. 18 m b. 20 m c. 90 m d. 100 m

Q3. The height of Anshul's house is:

- a. 20 m b. 40 m c. 50 m d. 100 m

Q 4. When the tower casts a shadow of 40 m, same time the length of the shadow of Anshul's house is:

- a. 16 m b. 40 m
c. 100 m d. None of these

Q 5. Which of the following similarity criterion does not exist?

- a. AA b. SAS c. SSS d. RHS

Solutions

1. Let $CD = h$ m be the height of the tower.
Let $BE = 20$ m be the height of Digvijay house and GF be the height of Anshul's house.

$\Delta ACD \sim \Delta ABE$

$\frac{AC}{AB} = \frac{CD}{EB}$

$\Rightarrow \frac{50}{10} = \frac{h}{20}$

$\Rightarrow h = 100$ m

So, option (d) is correct.

2. Given $AB = 18$ m. Let $AC = x$
In similar ΔABE and ΔACD

$\frac{AB}{AC} = \frac{BE}{CD} \Rightarrow \frac{18}{x} = \frac{20}{100}$

$\Rightarrow x = \frac{18 \times 100}{20} = 18 \times 5 = 90$ m

So, option (c) is correct.

3. Let height of Anshul's house be $GF = h_1$
Since, $\Delta HFG \sim \Delta HCD$

$\therefore \frac{HF}{HC} = \frac{FG}{CD} \Rightarrow \frac{20}{50} = \frac{h_1}{100}$

$h_1 = \frac{20 \times 100}{50} = 40$ m

So, option (b) is correct.

4. Given, $HC = 40$ cm
Let length of the shadow of Anshul's house be $HF = l$ m.

Since, $\Delta HFG \sim \Delta HCD$

$\therefore \frac{HF}{HC} = \frac{FG}{CD}$

$\Rightarrow \frac{l}{40} = \frac{40}{100}$

$\Rightarrow l = \frac{40 \times 40}{100} = 16$ m

So, option (a) is correct.

5. RHS similarity

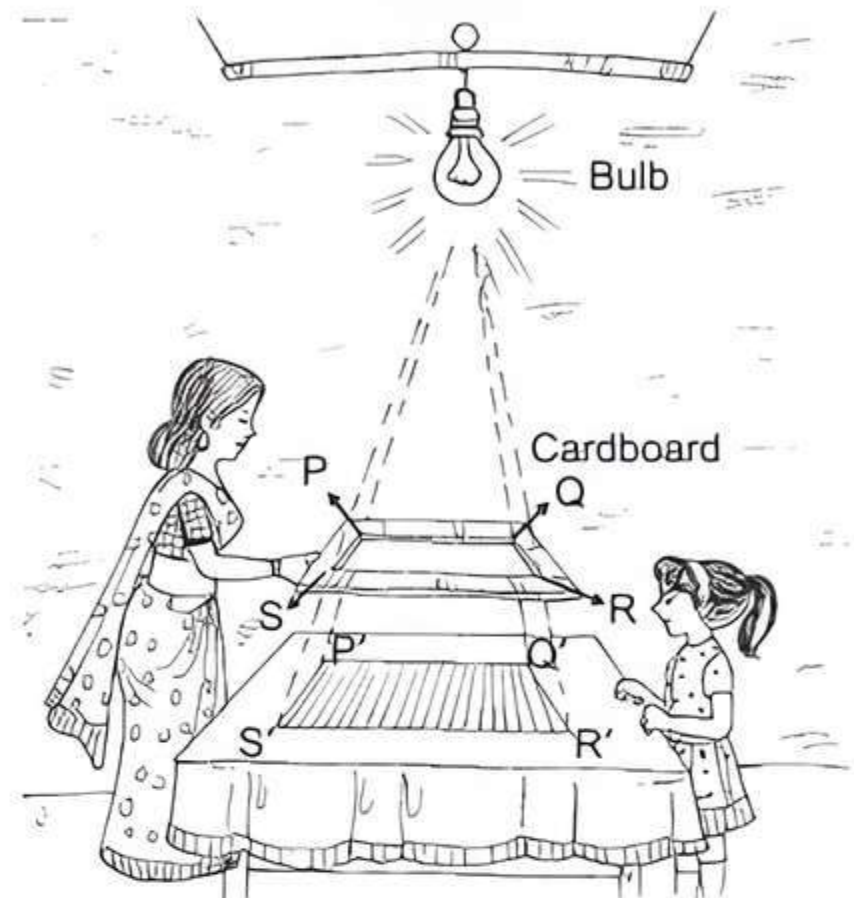
Criterion does not exist.

So, option (d) is correct.

Case Study 2

Gaurav placed a light bulb at a point O on the ceiling and directly below it placed a table. He cuts a polygon, say a quadrilateral PQRS, from a plane cardboard and place this cardboard

parallel to the ground between the lighted bulb and the table. Then a shadow of PQRS is cast on the table as P'Q'R'S'. Quadrilateral P'Q'R'S' is an enlargement of the quadrilateral PQRS with scale factor 1 : 3. Given that $PQ = 2.5$ cm, $QR = 3.5$ cm, $RS = 3.4$ cm and $PS = 3.1$ cm; $\angle P = 115^\circ$, $\angle Q = 95^\circ$, $\angle R = 65^\circ$ and $\angle S = 85^\circ$.



Based on the above information, solve the following questions:

Q 1. The length of R'S' is:

- a. 3.4 cm b. 10.2 cm c. 6.8 cm d. 9.5 cm

Q 2. The ratio of sides P'Q' and Q'R' is:

- a. 5 : 7 b. 7 : 5 c. 7 : 2 d. 2 : 7

Q 3. The measurement of $\angle Q'$ is:

- a. 115° b. 95° c. 65° d. 85°

Q 4. The sum of the lengths Q'R' and P'S' is:

- a. 12.3 cm b. 6.7 cm c. 19.8 cm d. 9 cm

Q 5. The sum of angles of quadrilateral P'Q'R'S' is:

- a. 180° b. 270° c. 300° d. 360°

Solutions

1. Given, scale factor is 1 : 3.

$\therefore R'S' = 3RS$

$\therefore R'S' = 3 \times 3.4 = 10.2$ cm

So, option (b) is correct.

2. Since, $P'Q' = 3PQ = 3 \times 2.5 = 7.5$ cm

and $Q'R' = 3QR = 3 \times 3.5 = 10.5$ cm

$\therefore \frac{P'Q'}{Q'R'} = \frac{7.5}{10.5} = \frac{5}{7}$ or 5 : 7

So, option (a) is correct.

3. Quadrilateral P'Q'R'S' is similar to PQRS

$\therefore \angle Q' = \angle Q = 95^\circ$

So, option (b) is correct.

4. $Q'R' = 3QR = 3 \times 3.5 = 10.5$ cm

and $P'S' = 3PS = 3 \times 3.1 = 9.3$ cm

$\therefore Q'R' + P'S' = 10.5 + 9.3 = 19.8$ cm

So, option (c) is correct.

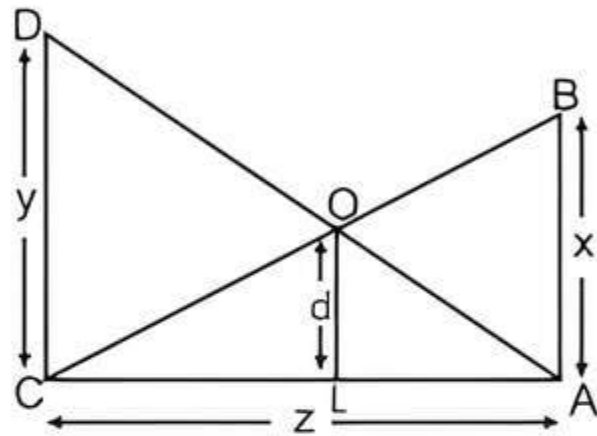
5. Since, PQRS ~ P'Q'R'S'
 $\therefore \angle P' = \angle P = 115^\circ$
 $\angle Q' = \angle Q = 95^\circ$
 $\angle R' = \angle R = 65^\circ$
 and $\angle S' = \angle S = 85^\circ$
 $\therefore \angle P' + \angle Q' + \angle R' + \angle S' = 115^\circ + 95^\circ + 65^\circ + 85^\circ$
 $= 360^\circ$

i.e., the sum of angles of quadrilateral P'Q'R'S' is 360° .

So, option (d) is correct.

Case Study 3

Anika is studying in class X. She observe two poles DC and BA. The heights of these poles are x m and y m respectively as shown in figure:



These poles are z m apart and O is the point of intersection of the lines joining the top of each pole to the foot of opposite pole and the distance between point O and L is d . Few questions came to his mind while observing the poles.

Based on the above information, solve the following questions:

- Q1. Which similarity criteria is applicable in $\triangle CAB$ and $\triangle CLO$?
- Q2. If $CL = a$, then find a in terms of x, y and d .
- Q3. If $AL = b$, then find b in terms of x, y and d .

Solutions

1. In $\triangle CAB$ and $\triangle CLO$, we have
 $\angle CAB = \angle CLO = 90^\circ$
 $\angle C = \angle C$ (common)
 \therefore By AA similarity criterion,
 $\triangle CAB \sim \triangle CLO$
2. $\therefore \triangle CAB \sim \triangle CLO$



TIP

Corresponding sides of similar triangles are proportional.

$$\therefore \frac{CA}{CL} = \frac{AB}{LO} \Rightarrow \frac{z}{a} = \frac{x}{d} \Rightarrow a = \frac{zd}{x}$$

3. In $\triangle ALO$ and $\triangle ACD$,

We have

$$\angle ALO = \angle ACD = 90^\circ$$

$$\angle A = \angle A \quad (\text{common})$$

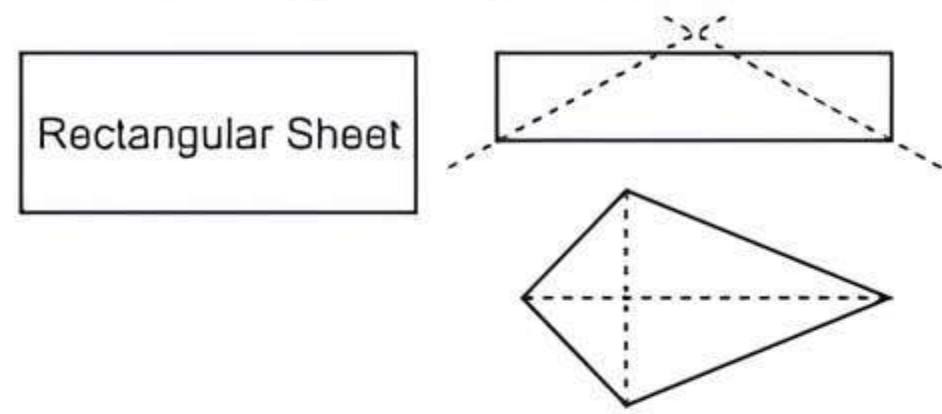
\therefore By AA similarity criterion,

$$\triangle ALO \sim \triangle ACD$$

$$\therefore \frac{AL}{AC} = \frac{OL}{DC} \Rightarrow \frac{b}{z} = \frac{d}{y} \Rightarrow b = \frac{zd}{y}$$

Case Study 4

Before Basant Panchami, Samarth is trying to make kites at home. So, he take a rectangular sheet and fold it horizontally, then vertically and fold it transversally. After cutting transversally, he gets a kite shaped figure as shown below:

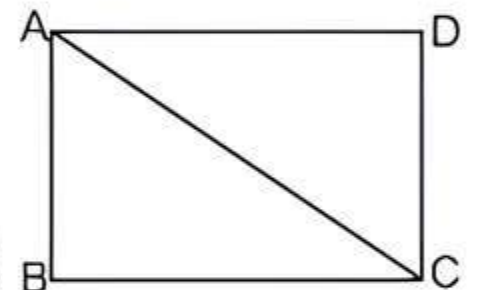


Based on the above information, solve the following questions:

- Q1. What is the angle between diagonals of a rectangle?
- Q2. Prove that two triangles divided by a diagonal in rectangle are similar as well as congruent.
- Q3. By which similarity criterion the triangles formed by longest diagonal in a kite are similar?

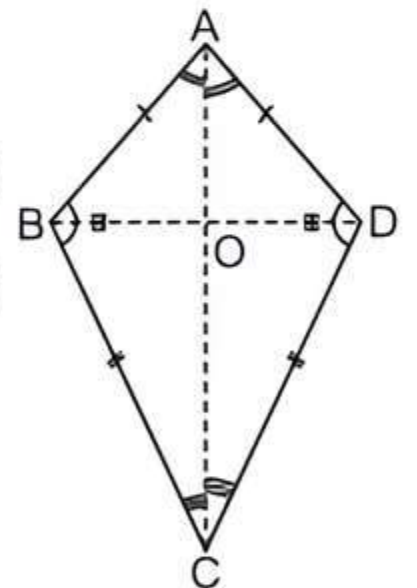
Solutions

1. Diagonals of a rectangle can bisect each other at any angle.
2. In $\triangle ABC$ and $\triangle CDA$
 $AB = CD$
 $\angle B = \angle D$
 $BC = DA$
 $\triangle ABC \cong \triangle CDA$
 (By SAS)



When two triangles are congruent, then they are similar also.

3. In $\triangle ABC$ and $\triangle ADC$,
 $AB = AD$
 $BC = DC$
 $AC = AC$ (common)
 $\therefore \triangle ABC \sim \triangle ADC$
 (by SSS criterion)



- In $\triangle ABC$ and $\triangle ADC$,
 $AB = AD$
 $\angle ABC = \angle ADC$
 $BC = DC$
 $\therefore \triangle ABC \sim \triangle ADC$
 (by SAS criterion)

- In $\triangle AOB$ and $\triangle AOD$,
 $AB = AD$
 $OA = OA$ (common)
 $BO = DO$
 (diagonal AC bisect the other diagonal BD)
 $\therefore \triangle AOB \sim \triangle AOD$ (by SSS similarity)

- $\Rightarrow \angle BAO = \angle DAO$
- In $\triangle BOC$ and $\triangle DOC$,
 $BC = DC$
 $OC = OC$ (common)
 $BO = OD$

- (Diagonal AC bisect the other diagonal BD)
 $\therefore \triangle BOC \sim \triangle DOC$ (by SSS similarity)

- $\Rightarrow \angle BCO = \angle DCO$

- In $\triangle ABC$ and $\triangle ADC$,
 $\angle B = \angle D$

$\angle BAC = \angle DAC$
 $(\because \angle BAO = \angle BAC, \angle DAO = \angle DAC, \text{proved above})$
 $\angle BCA = \angle DCA$
 $(\because \angle BCO = \angle BCA, \angle DCO = \angle DCA, \text{proved above})$
 $\therefore \triangle ABC \sim \triangle ADC$ (by AAA similarity)
 So, required similarity criterilons are SSS, SAS and AAA.

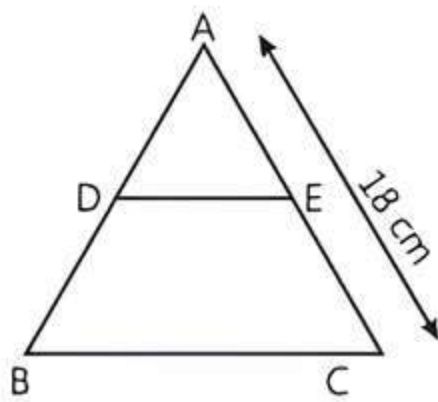
Very Short Answer Type Questions

Q1. In $\triangle PQR$, S and T are points on the sides PQ and PR respectively, such that $ST \parallel QR$. If $PS = 4$ cm, $PQ = 9$ cm and $PR = 4.5$ cm, then find PT.

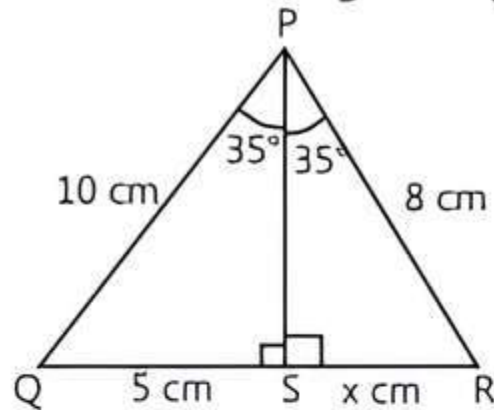
[CBSE 2015, 16, 17]

Q2. In two triangles ABC and DEF, if $\angle A = \angle E$ and $\angle B = \angle F$. Then, prove that $\frac{AB}{AC} = \frac{EF}{ED}$.

Q3. In the given figure, DE is parallel to BC. If $\frac{AD}{DB} = \frac{2}{3}$ and $AC = 18$ cm, then find AE.

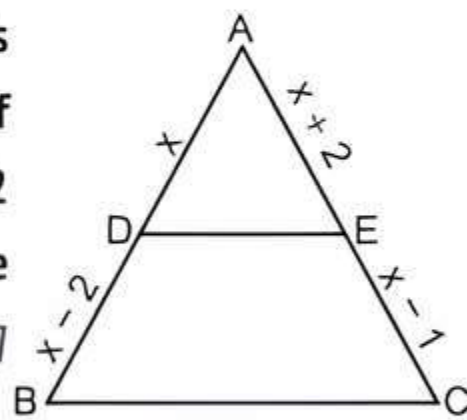


Q4. Find the value of x in the given figure.

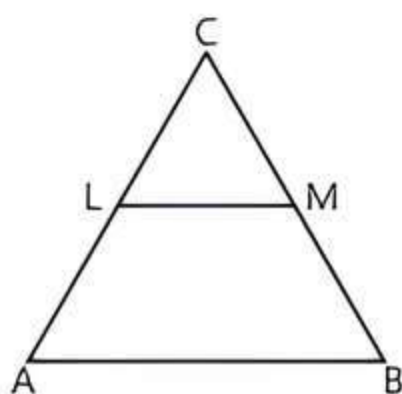


Q5. If the corresponding medians of two similar triangles are in the ratio 5 : 7, then find the ratio of their corresponding sides. [CBSE 2015]

Q6. In the given figure, ABC is a triangle which $DE \parallel BC$. If $AD = x$, $DB = x - 2$, $AE = x + 2$ and $EC = x - 1$, then find the value of x. [CBSE 2023]

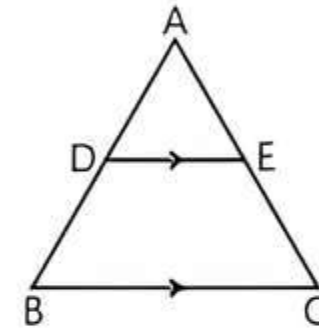


Q7. In the given figure, $LM \parallel AB$. If $AL = x - 3$, $AC = 2x$, $BM = x - 2$ and $BC = 2x + 3$, find the value of x.

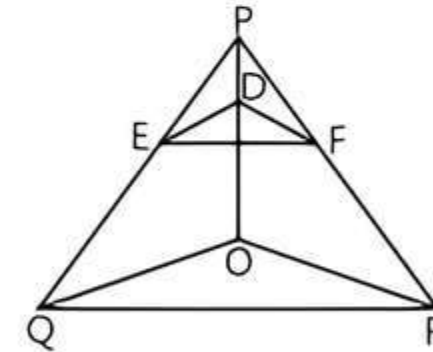


Short Answer Type-I Questions

Q1. In the given figure, a $\triangle ABC$, $DE \parallel BC$, so that $AD = (4x - 3)$ cm, $AE = (8x - 7)$ cm, $BD = (3x - 1)$ cm and $CE = (5x - 3)$ cm. Find the value of x. [CBSE 2015]

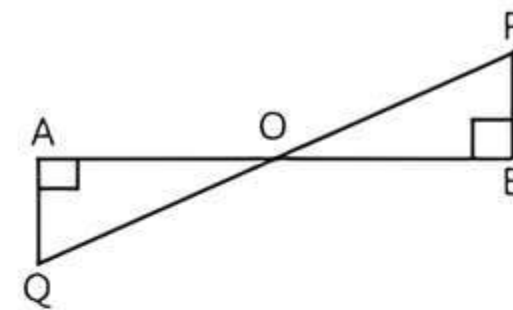


Q2. In the following figure, $DE \parallel OQ$ and $DF \parallel OR$, show that $EF \parallel QR$. [NCERT EXERCISE]

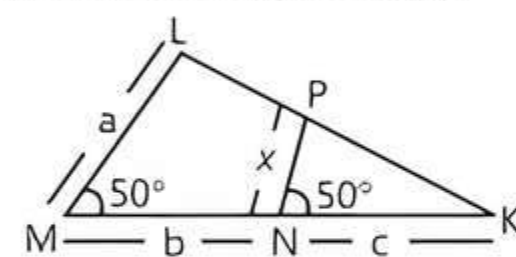


Q3. X and Y are points on the sides AB and AC respectively of a triangle ABC, such that $\frac{AX}{AB} = \frac{1}{4}$, $AY = 2$ cm and $YC = 6$ cm. Find whether $XY \parallel BC$ or not. [CBSE 2015]

Q4. In the given figure, $QA \perp AB$ and $PB \perp AB$. If $AO = 20$ cm, $BO = 12$ cm, $PB = 18$ cm, find AQ. [CBSE 2017]



Q5. In the given figure, $\angle M = \angle N = 50^\circ$. Express x in terms of a, b and c where a, b and c are the lengths of LM, MN and NK respectively.



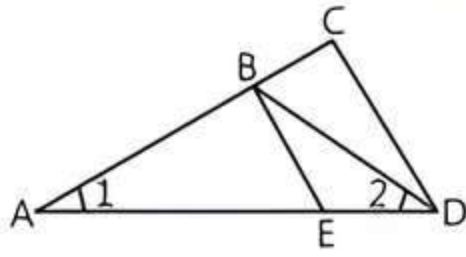
Q6. A vertical stick which is 15 cm long casts a 12 cm long shadow on ground. At the same time, a vertical tower casts a 50 m long shadow on the ground. Find the height of the tower. [CBSE 2016]

Q7. A girl of height 100 cm is walking away from the base of a lamp post at a speed of 1.9 m/s. If the lamp is 5 m above the ground, find the length of her shadow after 4s. [CBSE 2015]

Q8. ABCD is a trapezium such that $BC \parallel AD$ and $AB = 4$ cm. If the diagonals AC and BD intersect at O such that $\frac{AO}{OC} = \frac{OB}{OD} = \frac{1}{2}$, then find CD.

Q 9. In the given figure below, $\frac{AD}{AE} = \frac{AC}{BD}$ and $\angle 1 = \angle 2$.

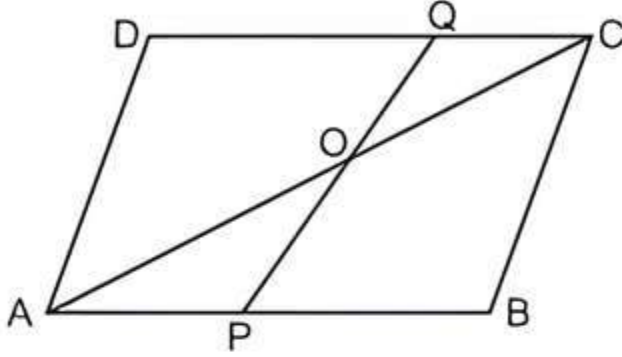
Show that $\triangle BAE \sim \triangle CAD$. [CBSE SQP 2022-23]



Q 10. ABCD is a parallelogram. Point P divide AB in the ratio 2 : 3 and point Q divides DC in the ratio 4 : 1.

Prove that OC is half of OA.

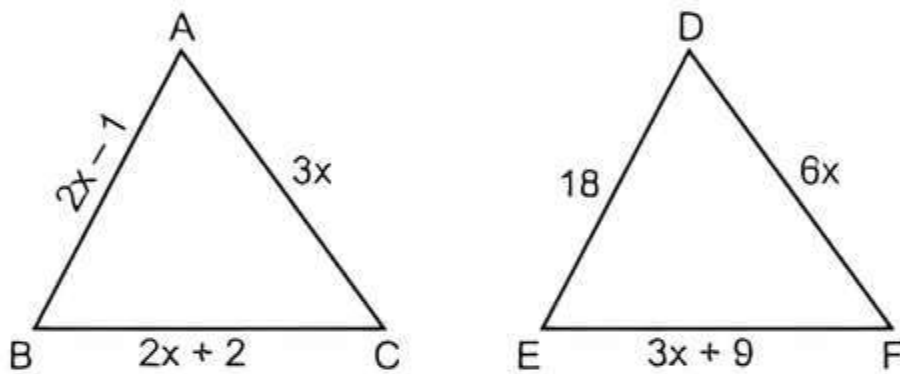
[CBSE SQP 2023-24]



Short Answer Type-II Questions

Q 1. In the given figure, if $\triangle ABC \sim \triangle DEF$ and their sides of lengths (in cm) are marked along them, then find the lengths of sides of each triangle.

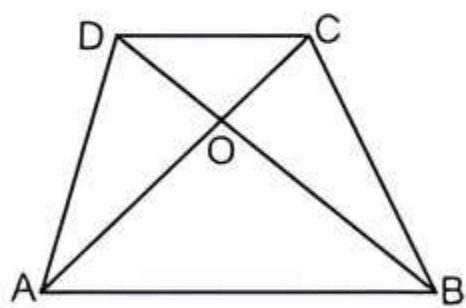
[NCERT EXEMPLAR; CBSE 2020]



Q 2. Diagonals AC and BD of trapezium ABCD with $AB \parallel DC$ intersect each other at point O. Show that

$$\frac{OA}{OC} = \frac{OB}{OD}$$

[CBSE 2023, NCERT EXERCISE]



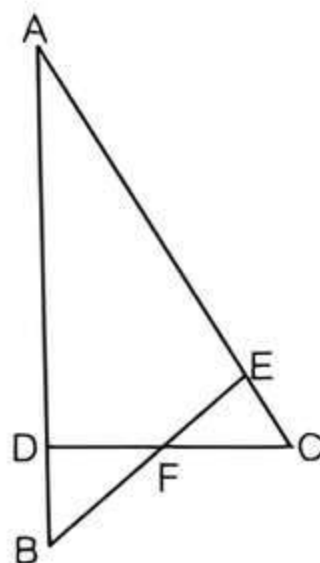
Q 3. The diagonals of a quadrilateral ABCD intersect each other at the point O such that $\frac{AO}{BO} = \frac{CO}{DO}$.

Show that ABCD is a trapezium. [NCERT EXERCISE]

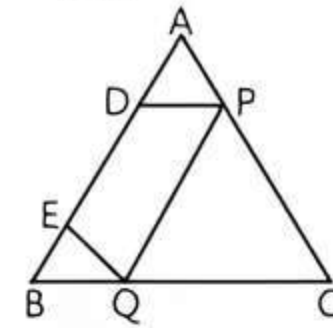
Q 4. In the given figure $\angle CEF = \angle CFE$. F is the mid-point of DC. Prove that

$$\frac{AB}{BD} = \frac{AE}{FD}$$

[CBSE SQP 2023-24]



Q 5. In the given figure, D and E are two points lying on side AB, such that $AD = BE$. If $DP \parallel BC$ and $EQ \parallel AC$, then prove that $PQ \parallel AB$.

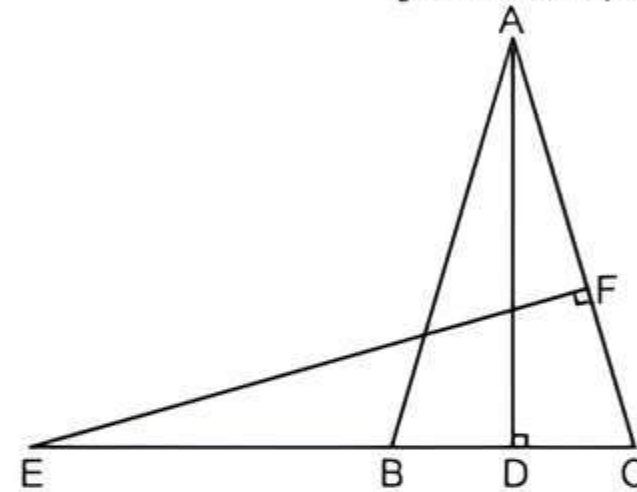


Q 6. If AD and PM are medians of triangles ABC and PQR, respectively, where $\triangle ABC \sim \triangle PQR$, prove that $\frac{AB}{PQ} = \frac{AD}{PM}$. [NCERT EXERCISE; CBSE 2017]

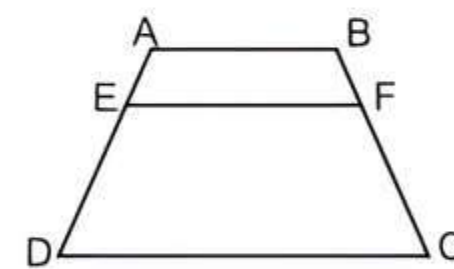
Q 7. If two poles 5 m and 15 m high are 100 m apart, then find the height of the point of intersection of the line joining the top of each pole to the foot of the opposite pole. [CBSE 2015]

Q 8. In the given figure, E is a point on the side CB produced of an isosceles triangle ABC with $AB = AC$. If $AD \perp BC$ and $EF \perp AC$, then prove that $\triangle ABD \sim \triangle ECF$.

[CBSE 2023, NCERT EXERCISE]



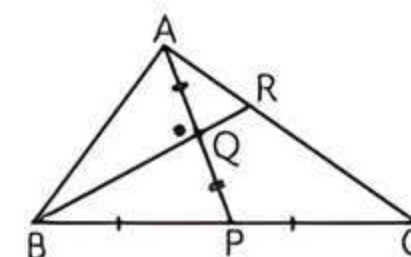
Q 9. In the given figure, $EF \parallel DC \parallel AB$. Prove that $\frac{AE}{ED} = \frac{BF}{FC}$.



Q 10. Through the mid-point M of the side CD of a parallelogram ABCD, the line BM is drawn intersecting AC in L and AD (produced) in E. Prove that $EL = 2BL$. [CBSE 2023]

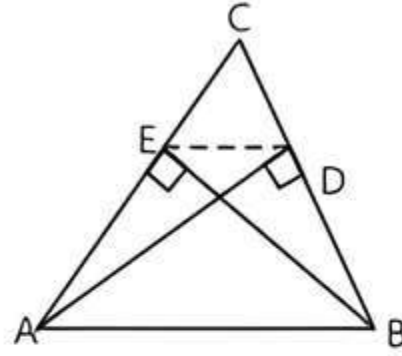
Long Answer Type Questions

Q 1. In the given figure of $\triangle ABC$, P is the mid-point of BC and Q is the mid-point of AP. If extended BQ meets AC in R, prove that $AR = \frac{1}{3}CA$. [CBSE 2016]



Q 2. Sides AB and AC and median AD of a $\triangle ABC$ are respectively proportional to sides PQ and PR and median PM of another $\triangle PQR$. Show that $\triangle ABC \sim \triangle PQR$. [NCERT EXERCISE; CBSE 2023, 17]

Q 3. In the given figure, AD and BE are respectively perpendiculars to BC and AC. Show that:

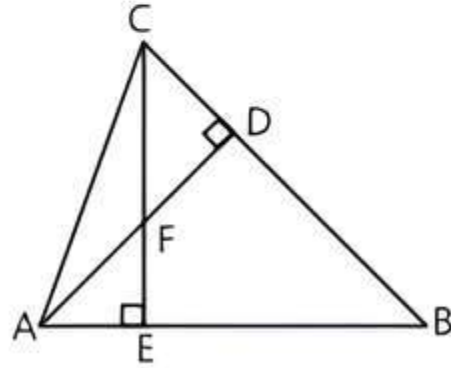


- (i) $\triangle ADC \sim \triangle BEC$ (ii) $CA \times CE = CB \times CD$
 (iii) $\triangle ABC \sim \triangle DEC$ (iv) $CD \times AB = CA \times DE$

Q 4. ABCD is a parallelogram, P is a point on side BC and DP when produced meets AB produced at L. Prove that:

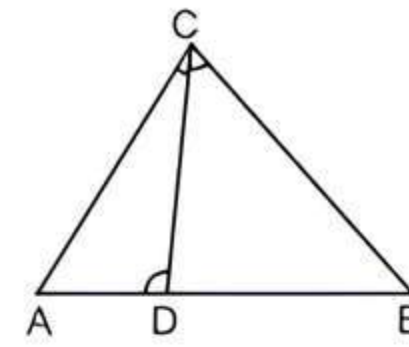
- (i) $\frac{DP}{PL} = \frac{DC}{BL}$ (ii) $\frac{DL}{DP} = \frac{AL}{DC}$

Q 5. In the given figure, AD and CE are two altitudes of $\triangle ABC$. Prove that:



- (i) $\triangle AEF \sim \triangle CDF$
 (ii) $\triangle ABD \sim \triangle CBE$
 (iii) $\triangle AEF \sim \triangle ADB$
 (iv) $\triangle FDC \sim \triangle BEC$ [NCERT EXERCISE]

Q 6. In the given figure, $\angle ADC = \angle BCA$; prove that $\triangle ACB \sim \triangle ADC$. Hence find BD if AC = 8 cm and AD = 3 cm. [CBSE SQP 2023-24, NCERT EXEMPLAR]



Q 7. In a $\triangle PQR$, N is a point on PR, such that $QN \perp PR$. If $PN \times NR = QN^2$, prove that $\angle PQR = 90^\circ$.

[CBSE SQP 2023-24, NCERT EXEMPLAR]

Q 8. Prove that if a line is drawn parallel to one side of a triangle intersecting the other two sides in distinct points, then the other two sides are divided in the same ratio. [CBSE 2023]

Using the above theorem, prove that a line through the point of intersection of the diagonals and parallel to the base of the trapezium divides the non-parallel sides in the same ratio.

[CBSE SQP 2022-23]

Or

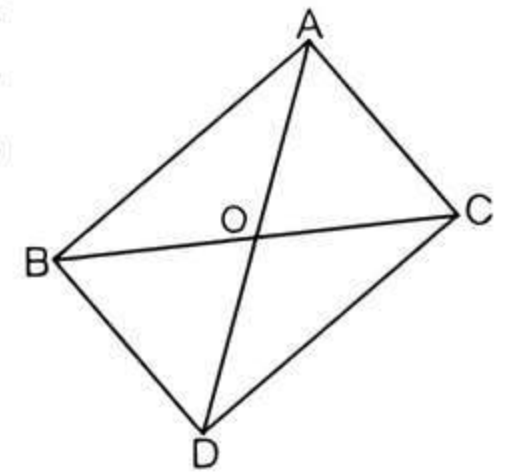
State and prove Basic Proportionality Theorem.

[CBSE SQP 2023-24]

Q 9. In the given figure, $\triangle ABC$ and $\triangle DBC$ are on the same base BC. If AD intersects BC at O, prove that

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{AO}{DO}$$

[NCERT EXEMPLAR; CBSE 2023]



Solutions

Very Short Answer Type Questions

1. Given, PS = 4 cm, PQ = 9 cm and PR = 4.5 cm

TIP

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Since, ST || QR, then by BPT, we have

$$\frac{PS}{PQ} = \frac{PT}{PR}$$

$$\Rightarrow \frac{4}{9} = \frac{PT}{4.5}$$

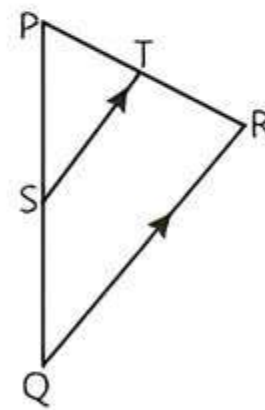
$$\text{or } PT = \frac{4 \times 4.5}{9} = 2 \text{ cm}$$

Hence, PT = 2 cm.

2. In $\triangle ABC$ and $\triangle EFD$,

$$\angle A = \angle E \text{ and } \angle B = \angle F \quad (\text{Given})$$

$$\therefore \triangle ABC \sim \triangle EFD \quad (\text{By AA similarity criterion})$$



$$\therefore \frac{AB}{EF} = \frac{AC}{ED} \Rightarrow \frac{AB}{AC} = \frac{EF}{ED} \quad \text{Hence proved.}$$

3. In $\triangle ABC$, DE || BC.

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \quad (\text{By Thales theorem})$$

$$\Rightarrow \frac{2}{3} = \frac{AE}{EC} \Rightarrow \frac{3}{2} = \frac{EC}{AE}$$

Adding 1 on both the sides, we get

$$\frac{3}{2} + 1 = \frac{EC}{AE} + 1 \Rightarrow \frac{3+2}{2} = \frac{EC+AE}{AE}$$

$$\Rightarrow \frac{5}{2} \frac{AC}{AE} \Rightarrow \frac{5}{2} = \frac{18}{AE}$$

$$\Rightarrow 5AE = 36$$

$$\Rightarrow AE = \frac{36}{5} \Rightarrow AE = 7.2 \text{ cm}$$

4. In $\triangle PSQ$ and $\triangle PSR$,

$$\angle QSP = \angle RSP = 90^\circ$$

$$\text{and } \angle QPS = \angle RPS = 35^\circ$$

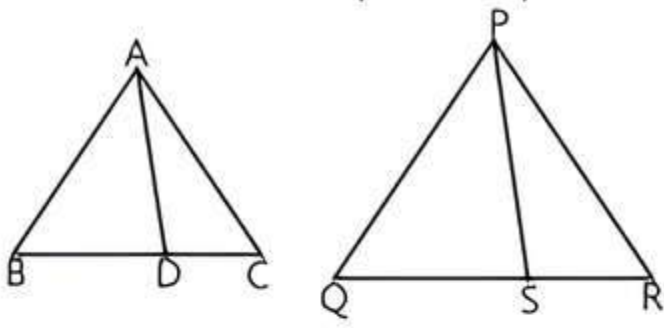


$$\triangle PSQ \sim \triangle PSR \quad (\text{By AA similarity})$$

$$\frac{PQ}{PR} = \frac{SQ}{SR}$$

$$\Rightarrow \frac{10}{8} = \frac{5}{x} \Rightarrow x = 4 \text{ cm}$$

5. Let ABC and PQR are two similar triangles with medians AD and PS respectively.



Then, $\frac{AD}{PS} = \frac{5}{7}$ (Given)

TR!CK

The ratio of the medians of two similar triangles is equal to the ratio of their corresponding sides.

$$\Rightarrow \frac{AB}{PQ} = \frac{AD}{PS} = \frac{5}{7} \quad (\because \triangle ABC \sim \triangle PQR)$$

Hence, the ratio of corresponding sides is 5 : 7.

6. In $\triangle ABC$, $DE \parallel BC$, so by BPT,

$$\frac{AD}{BD} = \frac{AE}{CE}$$

$$\Rightarrow \frac{x}{x-2} = \frac{x+2}{x-1}$$

$$\Rightarrow x(x-1) = (x-2)(x+2)$$

$$x^2 - x = x^2 - 4$$

$$\Rightarrow -x = -4 \Rightarrow x = 4$$

7.

TIP

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

In $\triangle ABC$, $LM \parallel AB$

$$\therefore \frac{AL}{LC} = \frac{BM}{MC} \quad (\text{By Thales theorem})$$

$$\Rightarrow \frac{AL}{AC - AL} = \frac{BM}{BC - BM}$$

$$\Rightarrow \frac{x-3}{2x-(x-3)} = \frac{x-2}{(2x+3)-(x-2)} \Rightarrow \frac{x-3}{x+3} = \frac{x-2}{x+5}$$

$$\Rightarrow (x-3)(x+5) = (x-2)(x+3)$$

$$\Rightarrow x^2 + 2x - 15 = x^2 + x - 6$$

$$\Rightarrow x = 9$$

Short Answer Type-I Questions

1. In $\triangle ABC$, $DE \parallel BC$, so by BPT,

$$\frac{AD}{BD} = \frac{AE}{CE} \Rightarrow \frac{4x-3}{3x-1} = \frac{8x-7}{5x-3}$$

$$\Rightarrow (4x-3)(5x-3) = (8x-7)(3x-1)$$

$$\Rightarrow 20x^2 - 12x - 15x + 9 = 24x^2 - 8x - 21x + 7$$

$$\Rightarrow 24x^2 - 20x^2 - 29x + 27x + 7 - 9 = 0$$

$$\Rightarrow 4x^2 - 2x - 2 = 0$$

$$\Rightarrow 2(2x^2 - x - 1) = 0$$

TR!CK

$\therefore 2 = 2 \times 1$
 \therefore Here, we have taken 2 and 1 as a factors of 2.
 So, middle term, $-1 = 1 - 2$.

$$\Rightarrow 2x^2 - 2x + x - 1 = 0$$

$$\Rightarrow 2x(x-1) + 1(x-1) = 0$$

$$\Rightarrow (x-1)(2x+1) = 0$$

$$\Rightarrow x-1 = 0 \text{ and } 2x+1 = 0$$

$$\Rightarrow x = 1 \text{ and } x = -\frac{1}{2}$$

When $x = -\frac{1}{2}$, then AD, BD, AE and CE all are negative.

$$\therefore x \neq -\frac{1}{2}$$

Hence, the value of x is 1.

COMMON ERROR

Sometimes students take both value of x as a answer but it is wrong. Students should cross check the value of x.

2.

TIP

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

In $\triangle POQ$, $DE \parallel OQ$

$$\therefore \frac{PE}{EQ} = \frac{PD}{DO} \quad (\text{By BPT}) \dots(1)$$

In $\triangle POR$, $DF \parallel OR$

$$\therefore \frac{PF}{FR} = \frac{PD}{DO} \quad (\text{By BPT}) \dots(2)$$

From eqs. (1) and (2), we get

$$\frac{PE}{EQ} = \frac{PF}{FR}$$

TR!CK

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

$\therefore EF \parallel QR$ (Converse of BPT) **Hence proved.**

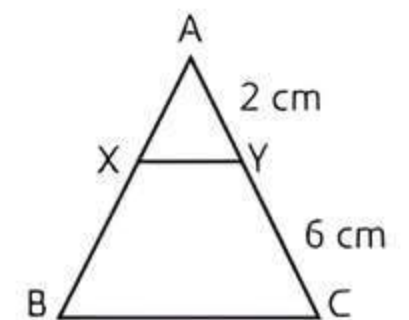
3. Given, $\frac{AX}{AB} = \frac{1}{4}$

Let $AX = k$, $AB = 4k$

$$\therefore XB = AB - AX = 4k - k = 3k$$

Now, $\frac{AX}{XB} = \frac{k}{3k} = \frac{1}{3}$

and $\frac{AY}{YC} = \frac{2}{6} = \frac{1}{3}$



TR!CK

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

$$\therefore \frac{AX}{XB} = \frac{AY}{YC}$$

Hence, $XY \parallel BC$ (By converse of BPT)

4. In $\triangle OAQ$ and $\triangle OBP$,

$$\angle OAQ = \angle OBP \quad (\text{Each } 90^\circ)$$

$$\angle AOQ = \angle BOP \quad (\text{Vertically opposite angles})$$

$$\therefore \triangle OAQ \sim \triangle OBP \quad (\text{By AA similarity})$$

$$\Rightarrow \frac{AO}{BO} = \frac{AQ}{PB} \quad (\text{Corresponding sides are proportional})$$

$$\Rightarrow \frac{20}{12} = \frac{AQ}{18}$$

$$\Rightarrow AQ = \frac{18 \times 20}{12} = 30 \text{ cm}$$

5. Given, $LM = a$, $PN = x$, $MN = b$ and $NK = c$

In $\triangle PNK$ and $\triangle LMK$,

$$\angle PNK = \angle LMK \quad (\text{Each } 50^\circ)$$

$$\angle PKN = \angle LKM \quad (\text{Common angle})$$

$$\therefore \triangle PNK \sim \triangle LMK \quad (\text{By AA similarity})$$

$$\text{So, } \frac{NK}{MK} = \frac{PN}{LM} \quad (\text{Corresponding sides are proportional})$$

$$\Rightarrow \frac{NK}{MN + NK} = \frac{PN}{LM}$$

$$\Rightarrow \frac{c}{b+c} = \frac{x}{a} \quad \text{or} \quad x = \frac{ac}{b+c}$$

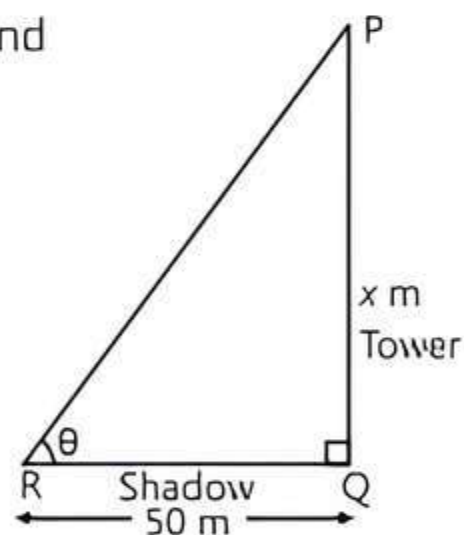
6. Let AB be the vertical stick and BC be its shadow.

Given, $AB = 15 \text{ cm} = 0.15 \text{ m}$

and $BC = 12 \text{ cm} = 0.12 \text{ m}$

Let PQ be the vertical tower and QR be its shadow.

In $\triangle ABC$ and $\triangle PQR$,



TIP

Two triangles are similar, if their corresponding sides are in proportional.

$$\angle ABC = \angle PQR \quad (\text{Each } 90^\circ)$$

$$\angle ACB = \angle PRQ$$

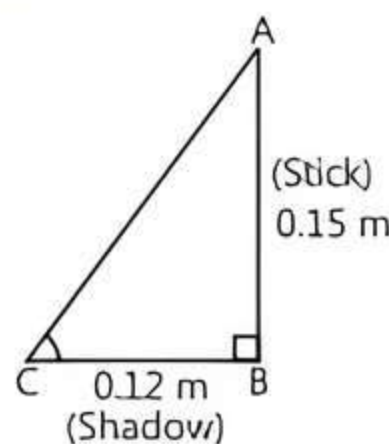
(Angular elevation of the sun at the same time)

$$\therefore \triangle ABC \sim \triangle PQR \quad (\text{By AA similarity})$$

$$\text{So, } \frac{AB}{PQ} = \frac{BC}{QR}$$

(Corresponding sides are proportional)

$$\Rightarrow \frac{0.15}{x} = \frac{0.12}{50}$$

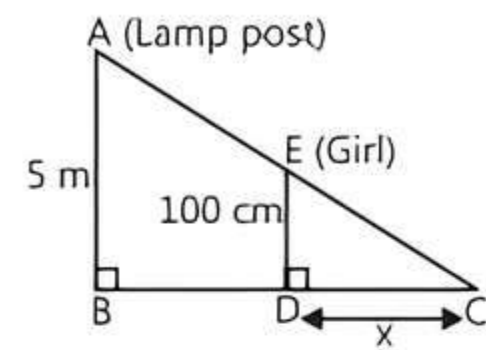


$$\Rightarrow x = \frac{0.15 \times 50}{0.12}$$

$$\text{or } x = 62.5 \text{ m}$$

Hence, the height of the tower is 62.5 m.

7. Let AB be the lamp post and ED be the position of girl after 4s.



Given, height of the girl $ED = 100 \text{ cm}$

and height of the lamp post $AB = 5 \text{ m} = 500 \text{ cm}$

Distance of the girl from lamp post after 4s

$$= 1.9 \times 4 = 7.6 \text{ m} = 760 \text{ cm}$$

(\because Distance = Speed \times Time)

$$\text{i.e., } BD = 760 \text{ cm}$$

$$\text{Let } DC = x \text{ cm}$$

TIP

Students should know about AAA criteria for similarity of triangles.

In $\triangle CDE$ and $\triangle CBA$,

$$\angle DCE = \angle BCA \quad (\text{Common angle})$$

$$\angle CDE = \angle CBA \quad (\text{Each } 90^\circ)$$

$$\therefore \triangle CDE \sim \triangle CBA \quad (\text{By AA similarity})$$

$$\text{So, } \frac{CD}{CB} = \frac{DE}{BA} \Rightarrow \frac{x}{x+760} = \frac{100}{500}$$

$$\Rightarrow 5x = x + 760 \Rightarrow 4x = 760$$

$$\Rightarrow x = 190 \text{ cm}$$

Hence, the length of her shadow after 4 s is 190 cm.

8.

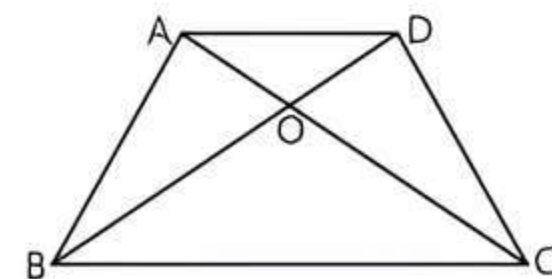
TIP

When two non-parallel rays intersect at a point, the angles formed between these rays at point of intersection in opposite directions are called vertically opposite angles (VOA).

In $\triangle AOB$ and $\triangle COD$, we have

$$\angle AOB = \angle COD \quad (\text{Vertically opposite angles})$$

$$\frac{AO}{OC} = \frac{OB}{OD} = \frac{1}{2} \quad (\text{Given})$$



$$\therefore \triangle AOB \sim \triangle COD \quad (\text{By SAS similarity criterion})$$

$$\therefore \frac{AB}{CD} = \frac{AO}{OC} \Rightarrow \frac{4}{CD} = \frac{1}{2} \Rightarrow CD = 8 \text{ cm}$$

9. In $\triangle ABD$,

$$\angle 1 = \angle 2 \quad (\text{Given})$$

$$\therefore BD = AB \quad (\text{Sides opposite to equal angles are equal}) \dots(1)$$

Given,

$$\frac{AD}{AE} = \frac{AC}{BD}$$

Using eq. (1),

$$\frac{AD}{AE} = \frac{AC}{AB} \quad \dots(2)$$

In $\triangle BAE$ and $\triangle CAD$,

$$\frac{AC}{AB} = \frac{AD}{AE} \quad (\text{From eq. (2)})$$

$$\angle A = \angle A \quad (\text{Common})$$

$$\triangle BAE \sim \triangle CAD \quad (\text{By SAS similarity criterion})$$

Hence proved.

10. Given ABCD is a parallelogram.

Let $AB = DC = a$

Now point P divides AB in the ratio 2 : 3

$$\therefore AP = \frac{2}{2+3} \cdot a = \frac{2a}{5} \text{ and } BP = \frac{3}{2+3} \cdot a = \frac{3a}{5}$$

Again, point Q divides DC in the ratio 4 : 1.

$$\therefore DQ = \frac{4}{4+1} \cdot a = \frac{4a}{5} \text{ and } CQ = \frac{1}{4+1} \cdot a = \frac{a}{5}$$

In $\triangle APO$ and $\triangle CQO$

$$\angle AOP = \angle COQ \quad (\text{vertically opposite angles})$$

$$\angle OAP = \angle OCQ$$

(In parallelogram, $AB \parallel CD$ and AC is transverse)

$$\therefore \triangle APO \sim \triangle CQO$$

(By AA similarity criterion)

$$\Rightarrow \frac{AP}{CQ} = \frac{PO}{QO} = \frac{AO}{CO}$$

$$\Rightarrow \frac{2a/5}{a/5} = \frac{AO}{CO} \Rightarrow \frac{AO}{CO} = \frac{2a}{a} = \frac{2}{1}$$

$$\Rightarrow OC = \frac{1}{2}OA \quad \text{Hence proved.}$$

Short Answer Type-II Questions

1. Since, $\triangle ABC \sim \triangle DEF$, so ratio of their corresponding sides is equal.

So,

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\Rightarrow \frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{3x}{6x}$$

$$\Rightarrow \frac{2x-1}{18} = \frac{2x+2}{3x+9} = \frac{1}{2}$$

Taking first and third parts, we get

$$\frac{2x-1}{18} = \frac{1}{2}$$

$$\Rightarrow 2x = 9 + 1$$

$$\Rightarrow x = \frac{9+1}{2} = \frac{10}{2} = 5$$

\therefore Sides of $\triangle ABC$.

$$AB = (2x - 1) = 2 \times 5 - 1 = 10 - 1 = 9 \text{ cm.}$$

$$BC = 2x + 2 = 2 \times 5 + 2 = 10 + 2 = 12 \text{ cm}$$

$$\text{and } AC = 3x = 3 \times 5 = 15 \text{ cm}$$

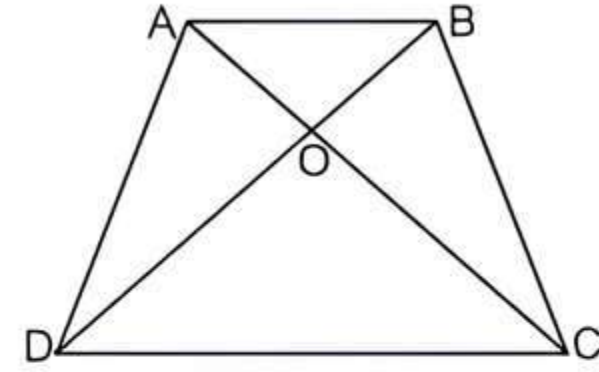
\therefore Sides of $\triangle DEF$,

$$DE = 18 \text{ cm,}$$

$$EF = 3 \times 5 + 9 = 15 + 9 = 24 \text{ cm}$$

$$\text{and } DF = 6x = 6 \times 5 = 30 \text{ cm}$$

2. Given: ABCD is a trapezium in which $AB \parallel CD$ and its diagonals AC and BD intersect at point O.



$$\text{To Prove: } \frac{OA}{OC} = \frac{OB}{OD}$$

Proof : $\because AB \parallel CD$ and AC is a transversal.

$$\therefore \angle OAB = \angle OCD \quad (\text{alternate angles})$$

$$\text{and } \angle AOB = \angle COD$$

(vertically opposite angles)

Now, In $\triangle AOB$ and $\triangle OCD$,

$$\angle AOB = \angle COD \text{ and } \angle OAB = \angle OCD$$

From AA similarity,

$$\triangle AOB \sim \triangle OCD$$

$$\frac{OA}{OC} = \frac{OB}{OD} \quad (\text{From the proportionality of side})$$

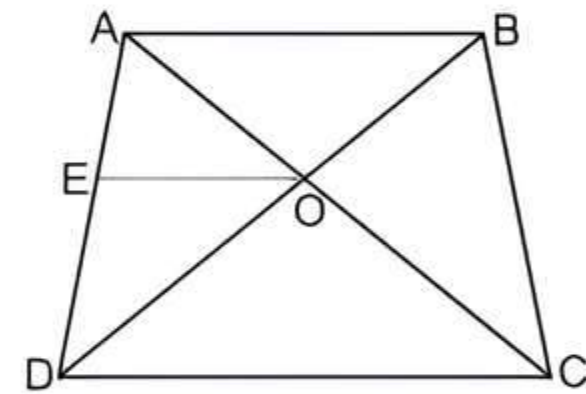
Hence proved.

3. Given : In a quadrilateral ABCD, $\frac{AO}{BO} = \frac{CO}{DO}$

To Prove : ABCD is a trapezium.

Construction : Let us draw a quadrilateral ABCD.

Draw a line $OE \parallel AB$.



Proof : In $\triangle ABD$, $OE \parallel AB$



TIP

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

By using basic proportionality theorem, we get

$$\frac{AE}{ED} = \frac{BO}{OD} \quad \dots(1)$$

But, it is given that

$$\frac{AO}{BO} = \frac{CO}{DO}$$

$$\Rightarrow \frac{AO}{OC} = \frac{BO}{OD} \quad \dots(2)$$

From eqs. (1) and (2), we get

$$\frac{AE}{ED} = \frac{AO}{OC}$$

TR!CK

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

⇒ $EO \parallel DC$ (By the converse of basic proportionality theorem)

⇒ $AB \parallel OE \parallel DC$

⇒ $AB \parallel DC$

∴ ABCD is a trapezium. **Hence proved.**

4. **Given:** In $\triangle CAD$, BE intersects CD at F, F is the mid-point of CD and $\angle CFE = \angle CEF$.

To Prove: $\frac{AB}{BD} = \frac{AE}{ED}$

Construction: Draw a line DG parallel to BE.

Proof: Since, F is the mid-point of DC, so $CF = FD$ and $\angle CEF = \angle CFE$ (Given)

∴ $CF = CE$

(∵ Sides opposite to equal angles are equal)

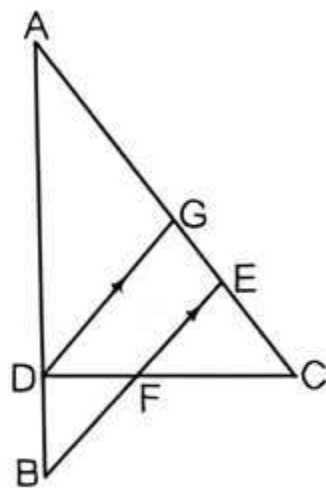
Now, in $\triangle CDG$, $FE \parallel DG$

⇒ $\frac{CE}{GE} = \frac{CF}{DF}$ (By BPT)

⇒ $\frac{CE}{GE} = \frac{CE}{DF}$ (∵ $CF = CE$)

$GE = DF$... (1)

In $\triangle ABE$, $DG \parallel BE$



⇒ $\frac{AB}{DB} = \frac{AE}{GE}$ (By BPT)

⇒ $\frac{AB}{DB} = \frac{AE}{DF}$ (From eq. (1))

or $\frac{AB}{BD} = \frac{AE}{FD}$ **Hence proved.**

5. **Given:** $AD \parallel BE$, $DP \parallel BC$ and $EQ \parallel AC$

To Prove: $PQ \parallel AB$

TIP

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

Proof: In $\triangle ABC$, by BPT we have

$$\frac{AD}{DB} = \frac{AP}{PC} \quad (\because DP \parallel BC) \dots(1)$$

Again, in $\triangle ABC$, by BPT we have

$$\frac{BE}{EA} = \frac{BQ}{QC} \quad (\because EQ \parallel AC)$$

or $\frac{AD}{DB} = \frac{BQ}{QC} \dots(2)$

(∵ $AD = BE$ and $EA = ED + DA = ED + BE = DB$)

TR!CK

If a line divides any two sides of a triangle in the same ratio, then the line is parallel to the third side.

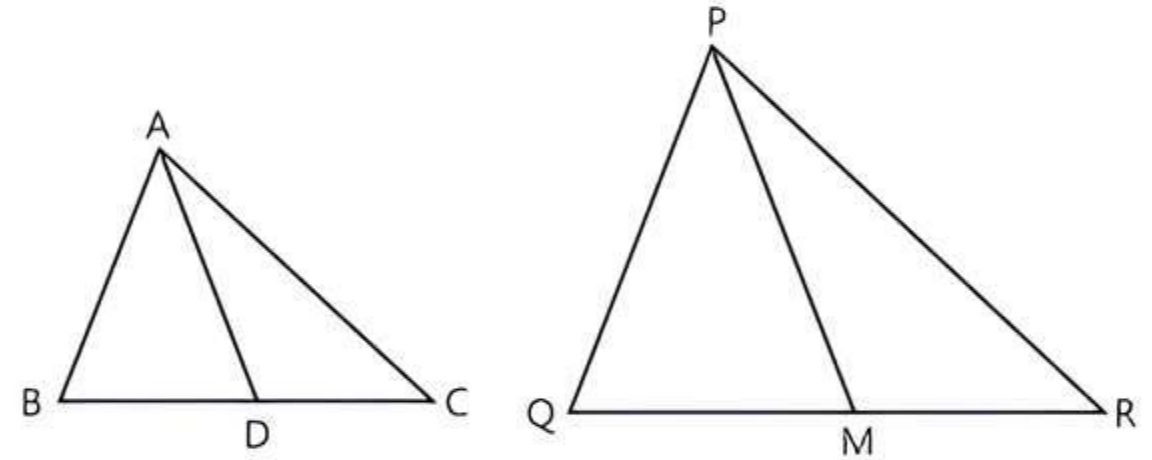
From eqs. (1) and (2), we get

$$\frac{AP}{PC} = \frac{BQ}{QC}$$

In $\triangle ABC$, P and Q divide the sides CA and CB respectively in the same ratio.

∴ $PQ \parallel AB$ **Hence proved.**

6. **Given:** $\triangle ABC \sim \triangle PQR$



To Prove: $\frac{AB}{PQ} = \frac{AD}{PM}$

Proof: Since, $\triangle ABC \sim \triangle PQR$

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{BC}{QR} \dots(1)$$

(Corresponding sides are proportional)

Also, $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$... (2)

(Corresponding angles are equal)

Since, AD and PM are medians, they will divide their opposite sides.

$$\therefore BD = \frac{BC}{2} \text{ and } QM = \frac{QR}{2} \dots(3)$$

From eqs. (1) and (3), we get

$$\frac{AB}{PQ} = \frac{BD}{QM} \dots(4)$$

Now, in $\triangle ABD$ and $\triangle PQM$,

$\angle B = \angle Q$ (Using eq. (2))

$$\frac{AB}{PQ} = \frac{BD}{QM} \text{ (Using eq. (4))}$$

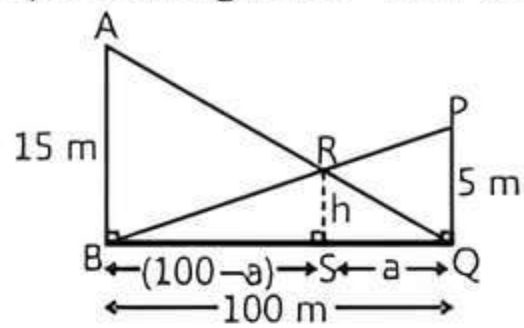
$\triangle ABD \sim \triangle PQM$ (By SAS similarity)

$$\Rightarrow \frac{AB}{PQ} = \frac{BD}{QM} = \frac{AD}{PM}$$

(Corresponding sides are proportional)

Hence proved.

7. Let the required height $RS = h$ m and $SQ = a$



$$\therefore BS = 100 - a \quad (\because BQ = 100 \text{ m})$$

$AB \perp BQ$ and $RS \perp BQ$

$\Rightarrow RS \parallel AB$

In $\triangle ABQ$ and $\triangle RSQ$,

$\angle AQB = \angle RQS$ (Common angle)

$\angle QBA = \angle QSR$ (Corresponding angles)

$\therefore \triangle ABQ \sim \triangle RSQ$ (By AA similarity)

$$\text{So, } \frac{AB}{RS} = \frac{BQ}{SQ}$$

(Corresponding sides are proportional)

$$\Rightarrow \frac{15}{h} = \frac{100}{a} \Rightarrow a = \frac{20h}{3} \quad \dots(1)$$

Now, in $\triangle RSB$ and $\triangle PQB$,

$\angle RBS = \angle PBQ$ (Common angle)

$\angle RSB = \angle PQB$ (Corresponding angles)

$\triangle RSB \sim \triangle PQB$ (By AA similarity)

$$\text{So, } \frac{RS}{PQ} = \frac{BS}{BQ} \quad (\text{Corresponding sides are proportional})$$

$$\Rightarrow \frac{h}{5} = \frac{100 - a}{100} \Rightarrow 20h = 100 - a$$

$$\Rightarrow 20h = 100 - \frac{20h}{3} \quad (\text{From eq. (1)})$$

$$\Rightarrow 20h + \frac{20h}{3} = 100$$

$$\Rightarrow \frac{80h}{3} = 100$$

$$\Rightarrow h = \frac{300}{80} = \frac{15}{4}$$

Hence, the required height is $\frac{15}{4}$ m.

8. It is given that ABC is an isosceles triangle.

$$\therefore AB = AC$$

$$\angle ABD = \angle ECF$$

(\because Angles opposite to equal sides are equal)

In $\triangle ABD$ and $\triangle ECF$

$\angle ADB = \angle EFC$ (Each 90°)

$\angle ABD = \angle ECF$ (Proved above)

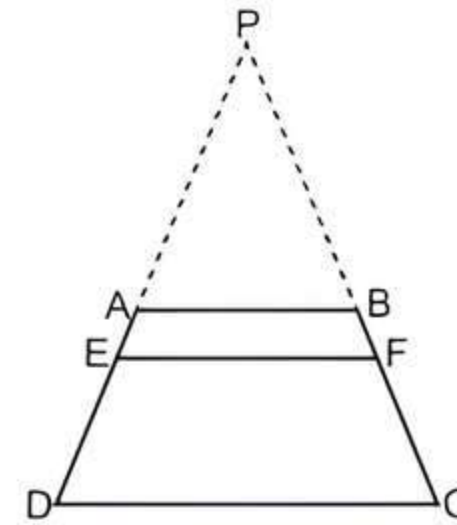
$\therefore \triangle ABD \sim \triangle ECF$ (By AA similarity) **Proved.**

9. Given: In the given figure, $EF \parallel DC \parallel AB$.

$$\text{To Prove: } \frac{AE}{ED} = \frac{BF}{FC}$$

Construction: Produce DA and CB to meet at P (say).

Proof: In $\triangle PEF$, we have



$AB \parallel EF$

$$\frac{PA}{AE} = \frac{PB}{BF} \quad (\text{By Thales theorem})$$

$$\Rightarrow \frac{PA}{AE} + 1 = \frac{PB}{BF} + 1$$

$$\Rightarrow \frac{PA + AE}{AE} = \frac{PB + BF}{BF}$$

$$\Rightarrow \frac{PE}{AE} = \frac{PF}{BF} \quad \dots(1)$$

In $\triangle PDC$, we have

$EF \parallel DC$

$$\frac{PE}{ED} = \frac{PF}{FC} \quad \dots(2)$$

(By Basic Proportionality Theorem)

On dividing eq. (1) by eq. (2), we get

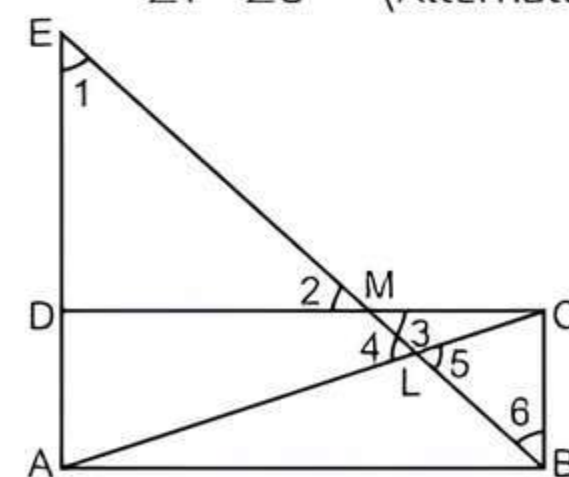
$$\frac{\frac{PE}{AE}}{\frac{PE}{ED}} = \frac{\frac{PF}{BF}}{\frac{PF}{FC}}$$

$$\Rightarrow \frac{ED}{AE} = \frac{FC}{BF}$$

$$\text{or } \frac{AE}{ED} = \frac{BF}{FC} \quad \text{Hence proved.}$$

10. In $\triangle DEM$ and $\triangle CBM$

$\angle 1 = \angle 6$ (Alternate interior angles)



$\angle 2 = \angle 3$ (Vertically opposite angle)

$DM = MC$ (M is the mid-point of CD)

$\triangle DEM \cong \triangle CBM$ (By AAS congruence criterion)

So, $DE = BC$ (By CPCT)

Also, $AD = BC$ (Opposite sides of a parallelogram)

$$\Rightarrow AE = AD + DE = 2BC$$

Now, $\angle 1 = \angle 6$ and $\angle 4 = \angle 5$
 $\triangle ELA \sim \triangle BLC$ (By AA similarity)

$$\Rightarrow \frac{EL}{BL} = \frac{EA}{BC}$$

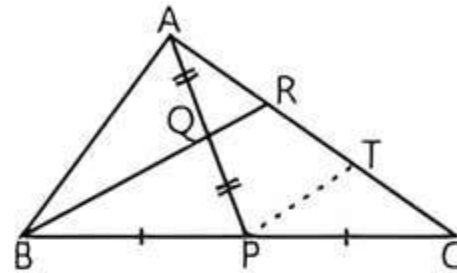
$$\Rightarrow \frac{EL}{BL} = \frac{2BC}{BC} = \frac{2}{1} \Rightarrow EL = 2BL \quad \text{Hence proved.}$$

Long Answer Type Questions

1. **Given:** In $\triangle ABC$, P is the mid-point of BC and Q is the mid-point of AP.

To Prove: $RA = \frac{1}{3}CA$

Construction: Draw $PT \parallel BR$.



Proof: In $\triangle CBR$, $PT \parallel BR$

$$\frac{CT}{TR} = \frac{CP}{PB} \quad (\text{By BPT})$$

TIP

If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, then the other two sides are divided in the same ratio.

$$\Rightarrow \frac{CT}{TR} = 1 \quad (\because P \text{ is mid-point of } BC \text{ i.e., } PB = CP)$$

$$\Rightarrow CT = TR \quad \dots(1)$$

In $\triangle APT$, $QR \parallel PT$

$$\frac{AQ}{QP} = \frac{AR}{RT} \quad (\text{By BPT})$$

$$\Rightarrow 1 = \frac{AR}{RT} \quad (\because Q \text{ is mid-point of } AP \text{ i.e., } AQ = QP)$$

$$\Rightarrow AR = RT \quad \dots(2)$$

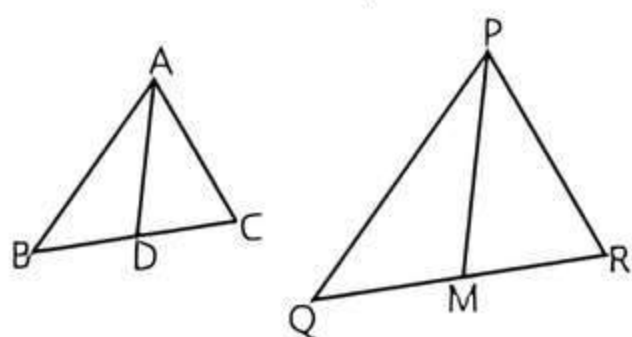
From eqs. (1) and (2), we get

$$AR = RT = CT$$

$$AR = \frac{1}{3}AC \quad \text{Hence proved.}$$

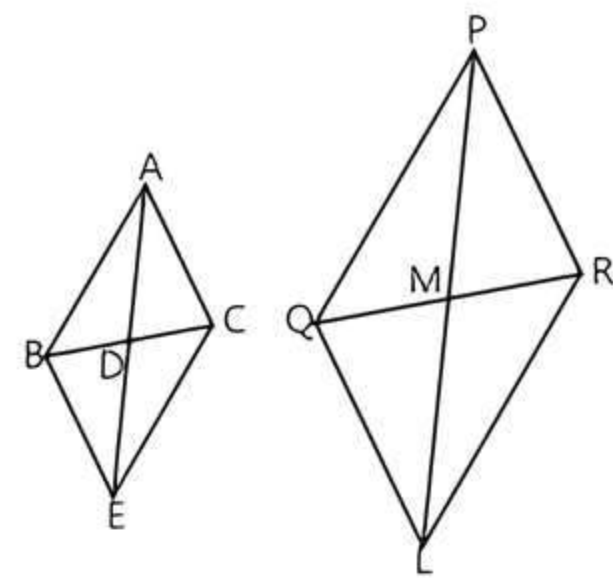
2. **Given:** $\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM}$

To Prove: $\triangle ABC \sim \triangle PQR$



Construction: Extend AD and PM up to point E and L respectively, such that $AD = DE$ and $PM = ML$.

Then join B to E, C to E, Q to L and R to L.



Proof: We know that, medians divide opposite sides.

Therefore, $BD = DC$ and $QM = MR$

Also, $AD = DE$ (By construction)

and $PM = ML$ (By construction)

In quadrilateral ABEC, diagonals AE and BC bisect each other at point D.

Therefore, quadrilateral ABEC is a parallelogram.

$$\therefore AC = BE \text{ and } AB = CE$$

(Opposite sides of a parallelogram are equal)

Similarly, quadrilateral PQLR is a parallelogram.

$$\therefore PR = QL \text{ and } PQ = RL$$

$$\frac{AB}{PQ} = \frac{AC}{PR} = \frac{AD}{PM} \quad (\text{Given})$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{2AD}{2PM} \quad (\text{Proved above})$$

$$\Rightarrow \frac{AB}{PQ} = \frac{BE}{QL} = \frac{AE}{PL}$$

TIP

If in two triangles, corresponding sides are in the same ratio, then their corresponding angles are equal and hence the triangles are similar (SSS).

$$\therefore \triangle ABE \sim \triangle PQL \quad (\text{By SSS similarity})$$

$$\Rightarrow \angle BAE = \angle QPL \quad \dots(1)$$

(Corresponding angles of similar triangles)

$$\text{Similarly, } \triangle AEC \sim \triangle PLR \Rightarrow \angle CAE = \angle RPL \quad \dots(2)$$

Adding eqs. (1) and (2), we get

$$\angle BAE + \angle CAE = \angle QPL + \angle RPL$$

$$\Rightarrow \angle CAB = \angle RPQ \quad \dots(3)$$

TIP

If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are in the same ratio, then the triangles are similar (SAS).

In $\triangle ABC$ and $\triangle PQR$,

$$\frac{AB}{PQ} = \frac{AC}{PR} \quad (\text{Given})$$

$$\angle CAB = \angle RPQ \quad (\text{Using eq. (3)})$$

$$\therefore \triangle ABC \sim \triangle PQR \quad (\text{By SAS similarity})$$

Hence proved.

3. (i) In $\triangle ADC$ and $\triangle BEC$, we have

$$\angle ADC = \angle BEC = 90^\circ \quad (\text{Given})$$

$$\angle ACD = \angle BCE \quad (\text{Common})$$

So, by AA criterion of similarity,

$$\triangle ADC \sim \triangle BEC$$

- (ii) We have,

$$\triangle ADC \sim \triangle BEC \quad (\text{As proved above})$$

$$\Rightarrow \frac{AC}{BC} = \frac{DC}{EC} \quad \dots(1)$$

$$\Rightarrow CA \times CE = CB \times CD$$

(iii) In $\triangle ABC$ and $\triangle DEC$, we have

$$\frac{AC}{BC} = \frac{DC}{EC} \quad \text{[From eq. (1)]}$$

$$\Rightarrow \frac{AC}{DC} = \frac{BC}{EC}$$

Also, $\angle ACB = \angle DCE$ (Common)

$\therefore \triangle ABC \sim \triangle DEC$

(iv) We have,

$\triangle ABC \sim \triangle DEC$ (As proved above)

$$\Rightarrow \frac{AB}{DE} = \frac{AC}{DC}$$

$$\Rightarrow AB \times DC = AC \times DE$$

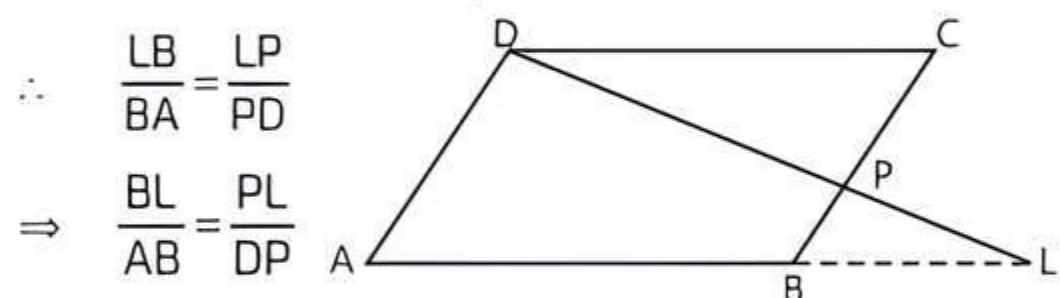
$$\Rightarrow CD \times AB = CA \times DE \quad \text{Hence proved.}$$

4. **Given:** A parallelogram ABCD in which P is a point on side BC such that DP produced meets AB produced at L.

To Prove: (i) $\frac{DP}{PL} = \frac{DC}{BL}$ (ii) $\frac{DL}{DP} = \frac{AL}{DC}$

Proof: (i) In $\triangle ALD$, we have

BP || AD



$$\therefore \frac{LB}{BA} = \frac{LP}{PD}$$

$$\Rightarrow \frac{BL}{AB} = \frac{PL}{DP}$$

$$\Rightarrow \frac{BL}{DC} = \frac{PL}{DP} \quad (\because AB = DC)$$

$$\Rightarrow \frac{DP}{PL} = \frac{DC}{BL} \quad \text{(Taking reciprocals of both sides)} \quad \text{Hence proved.}$$

(ii) From part (i), we have

$$\frac{DP}{PL} = \frac{DC}{BL}$$

$$\Rightarrow \frac{PL}{DP} = \frac{BL}{DC} \quad \text{(Taking reciprocals of both sides)}$$

$$\Rightarrow \frac{PL}{DP} = \frac{BL}{AB} \quad (\because DC = AB)$$

$$\Rightarrow \frac{PL}{DP} + 1 = \frac{BL}{AB} + 1$$

$$\Rightarrow \frac{DP + PL}{DP} = \frac{BL + AB}{AB}$$

$$\Rightarrow \frac{DL}{DP} = \frac{AL}{AB}$$

$$\Rightarrow \frac{DL}{DP} = \frac{AL}{DC} \quad (\because AB = DC) \quad \text{Hence proved.}$$

5. (i) In $\triangle AEF$ and $\triangle CDF$, we have

$$\angle AEF = \angle CDF = 90^\circ \quad (\because CE \perp AB \text{ and } AD \perp BC)$$

$$\angle AFE = \angle CFD \quad \text{(Vertically opposite angles)}$$

$\therefore \triangle AEF \sim \triangle CDF$ (By AA similarity)

(ii) In $\triangle ABD$ and $\triangle CBE$, we have
 $\angle ABD = \angle CBE = \angle B$ (Common angle)
 $\angle ADB = \angle CEB = 90^\circ$

($\because AD \perp BC$ and $CE \perp AB$)

$\therefore \triangle ABD \sim \triangle CBE$ (By AA similarity)

(iii) In $\triangle AEF$ and $\triangle ADB$, we have

$$\angle AEF = \angle ADB = 90^\circ$$

($\because AD \perp BC$ and $CE \perp AB$)

$$\angle FAE = \angle DAB \quad \text{(Common angle)}$$

$\therefore \triangle AEF \sim \triangle ADB$ (by AA similarity)

(iv) In $\triangle FDC$ and $\triangle BEC$, we have

$$\angle FDC = \angle BEC = 90^\circ$$

($\because AD \perp BC$ and $CE \perp AB$)

$$\therefore \angle FCD = \angle ECB \quad \text{(Common angle)}$$

$\therefore \triangle FDC \sim \triangle BEC$ (By AA similarity)

Hence proved.

6. Given $\angle BCA = \angle ADC$

In $\triangle ACB$ and $\triangle ADC$,

$$\angle BCA = \angle ADC \quad \text{(Given)}$$

$$\angle CAB = \angle DAC \quad \text{(Common angle)}$$

$\therefore \triangle ACB \sim \triangle ADC$ (By AA similarity)

Hence proved.

Also given, $AC = 8$ cm and $AD = 3$ cm.

We know that,

Sides of similar triangle are in same proportion.

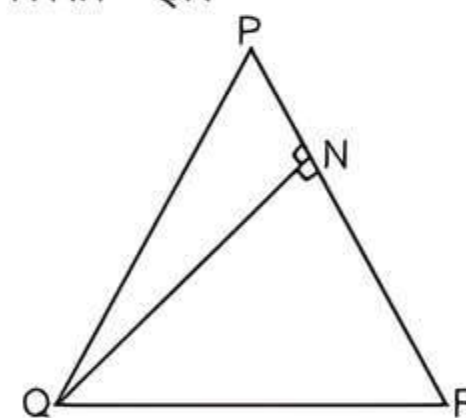
$$\therefore \frac{AC}{AD} = \frac{AB}{AC} \Rightarrow AC^2 = AB \times AD$$

$$AB = \frac{AC^2}{AD} = \frac{(8)^2}{3} = \frac{64}{3}$$

So,

$$BD = AB - AD = \frac{64}{3} - 3 = \frac{64 - 9}{3} = \frac{55}{3}$$

7. **Given:** $\triangle PQR$, N is a point on PR , such that $QN \perp PR$ and $PN \cdot NR = QN^2$



To Prove: $\angle PQR = 90^\circ$

Proof: We have $PN \cdot NR = QN^2$

$$\Rightarrow PN \cdot NR = QN \cdot QN$$

$$\Rightarrow \frac{PN}{QN} = \frac{QN}{NR} \quad \dots(1)$$

In $\triangle QNP$ and $\triangle RNQ$, $\frac{PN}{QN} = \frac{QN}{NR}$

and $\angle PNQ = \angle RNQ$ (Each equal to 90°)

$\therefore \triangle QNP \sim \triangle RNQ$ (By SAS similarity criterion)

Then $\triangle QNP$ and $\triangle RNQ$ are equiangular.

i.e. $\angle PQN = \angle QRN$

$$\angle RQN = \angle QPN$$

On adding both sides, we get

$$\angle PQN + \angle RQN = \angle QRN + \angle QPN$$

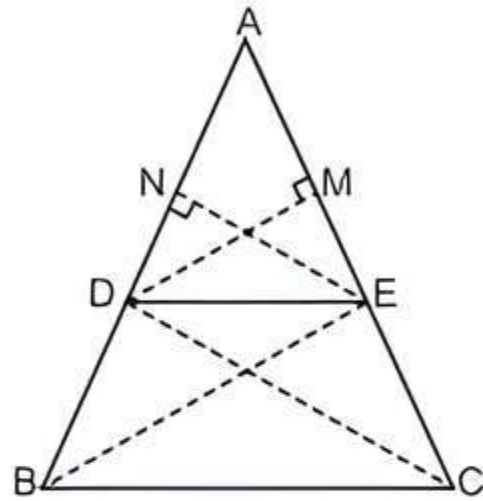
$$\Rightarrow \angle PQR = \angle QRN + \angle QPN \quad \dots(2)$$

We know that sum of angles of a triangle = 180°

In $\triangle PQR$, $\angle PQR + \angle QPR + \angle QRP = 180^\circ$
 $\Rightarrow \angle PQR + \angle QPN + \angle QRN = 180^\circ$
 $(\because \angle QPR = \angle QPN \text{ and } \angle QRP = \angle QRN)$
 (using Eq. (2))
 $\Rightarrow \angle PQR + \angle PQR = 180^\circ$
 $\Rightarrow 2 \angle PQR = 180^\circ$
 $\Rightarrow \angle PQR = \frac{180^\circ}{2} = 90^\circ$
 $\angle PQR = 90^\circ$ Hence proved.

8. **Given:** In $\triangle ABC$, $DE \parallel BC$

To Prove: $\frac{AD}{BD} = \frac{AE}{EC}$



Construction: Join BE and CD.

Draw $DM \perp AC$ and $EN \perp AB$

Proof: Here,

$$\text{ar}(\triangle ADE) = \frac{1}{2} \times AD \times EN$$

$$(\because \text{Area of triangle} = \frac{1}{2} \times \text{base} \times \text{height})$$

and $\text{ar}(\triangle BDE) = \frac{1}{2} \times DB \times EN$

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{\frac{1}{2} \times AD \times EN}{\frac{1}{2} \times DB \times EN} = \frac{AD}{DB} \quad \dots(1)$$

Also, $\text{ar}(\triangle ADE) = \frac{1}{2} \times AE \times DM$

and $\text{ar}(\triangle DEC) = \frac{1}{2} \times EC \times DM$

$$\frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DEC)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC}$$

Since, $\triangle BDE$ and $\triangle DEC$ are on the same base DE and between the same parallel lines BC and DE.

$$\therefore \text{ar}(\triangle BDE) = \text{ar}(\triangle DEC)$$

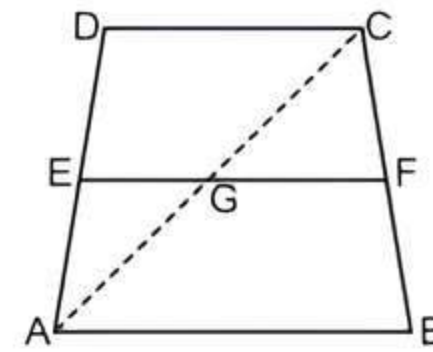
$$\therefore \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle DEC)} = \frac{\text{ar}(\triangle ADE)}{\text{ar}(\triangle BDE)} = \frac{AE}{EC} \quad \dots(2)$$

From eqs. (1) and (2),

$$\frac{AD}{DB} = \frac{AE}{EC} \quad \text{Hence proved.}$$

Let ABCD be a trapezium with $DC \parallel AB$ and EF be a line parallel to AB.

To Prove: $\frac{DE}{EA} = \frac{CF}{FB}$



Construction: Join AC, meeting EF in G.

Proof: Given, $AB \parallel DC$ and $EF \parallel AB$

So, $EF \parallel DC$ (Lines parallel to the same line are parallel to each other)

In $\triangle ABC$, $GF \parallel AB$ (As $EF \parallel DC$)

$$\frac{CG}{GA} = \frac{CF}{FB} \quad \text{(By BPT) } \dots(1)$$

In $\triangle ADC$, $EG \parallel DC$ (As $EF \parallel DC$)

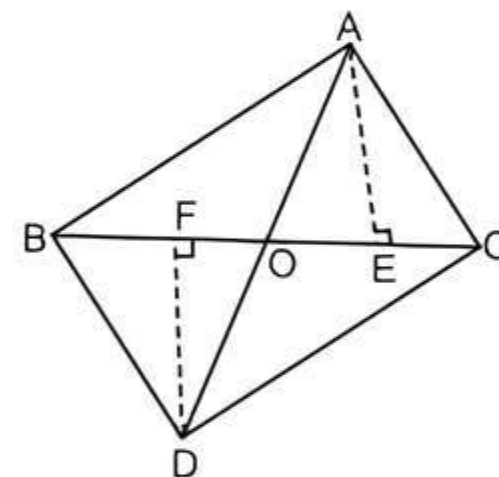
$$\frac{DE}{EA} = \frac{CG}{GA} \quad \text{(By BPT) } \dots(2)$$

From eqs. (1) and (2),

$$\frac{DE}{EA} = \frac{CF}{FB} \quad \text{Hence proved.}$$

9. **Given:** $\triangle ABC$ and $\triangle DBC$ are two triangles on same base BC. AC intersects BD at point O.

To Prove: $\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{AO}{DO}$



Construction: Draw the perpendicular AE from vertex A to BC and DF from vertex D to BC.

Proof: Since perpendiculars AD and DF are drawn from vertices A and D respectively to BC, therefore $\triangle AEO$ and $\triangle DFO$ are right angled.

In right angled $\triangle AEO$ and $\triangle DFO$,

$$\angle AEO = \angle DFO \quad (\text{each } 90^\circ)$$

$$\angle AOE = \angle DOF \quad (\text{vertically opposite angles})$$

$$\triangle AEO \sim \triangle DFO \quad (\text{by AA similarity})$$

$$\frac{AE}{DF} = \frac{AO}{DO} \quad \dots(1)$$

$$\text{Now, area of } \triangle ABC = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times BC \times AE$$

$$\text{and area of } \triangle DBC = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times BC \times DF$$

$$\frac{\text{area of } \triangle ABC}{\text{area of } \triangle DBC} = \frac{\frac{1}{2} \times BC \times AE}{\frac{1}{2} \times BC \times DF} = \frac{AE}{DF}$$

$$\Rightarrow \frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{AE}{DF} \quad \dots(2)$$

From eqs. (1) and (2), we get

$$\frac{\text{ar}(\triangle ABC)}{\text{ar}(\triangle DBC)} = \frac{AO}{DO} \quad \text{Hence proved.}$$



Chapter Test

Multiple Choice Questions

- Q 1. If in $\triangle ABC$, $AB = 6$ cm and $DE \parallel BC$ such that $AE = \frac{1}{4} AC$, then the length of AD is:
- a. 2 cm b. 12 cm
c. 1.5 cm d. 4 cm
- Q 2. In $\triangle PQR$ and $\triangle MNS$, $\frac{PQ}{NS} = \frac{QR}{MS} = \frac{PR}{MN}$, then symbolically we write as:
- a. $\triangle QRP \sim \triangle SMN$ b. $\triangle PQR \sim \triangle SMP$
c. $\triangle PQR \sim \triangle MNS$ d. None of these

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): In the following questions, a statement of Assertion (A) is followed by a statement of Reason (R). Choose the correct option:

- a. Both Assertion (A) and Reason (R) are true and reason (R) is the correct explanation of Assertion (A)
b. Both Assertion (A) and Reason (R) are true but reason (R) is not the correct explanation of Assertion (A)
c. Assertion (A) is true but Reason (R) is false
d. Assertion (A) is false but Reason (R) is true
- Q 3. **Assertion (A):** ABC is a triangle in which $AB = AC$ and D is a point on AC such that $BC^2 = AC \times CD$. Then $\triangle ABC \sim \triangle BDC$ by SAS similarity criterion.

Reason (R): If two angles of one triangle are respectively equal to the two angles of another triangle, then the two triangles are similar. This is known as SAS similarity criterion.

- Q 4. **Assertion (A):** In a $\triangle ABC$, D and E are points on sides AB and AC respectively, such that $BD = CE$. If $\angle B = \angle C$, then DE is not parallel to BC .
- Reason (R):** If a line divides any two sides of a triangle in the same ratio, then the line must be parallel to the third side.

Fill in the Blanks

- Q 5. Two polygons of the same number of sides are similar, if their corresponding angles are and their corresponding sides are, in the ratio.
- Q 6. All equilateral triangles are (similar/not similar).

True/False

- Q 7. Two figures having the same shapes is said to be similar figures.
- Q 8. In two triangles, if one pair of the corresponding sides are proportional and the included angles are also equal, then two triangles are not similar.

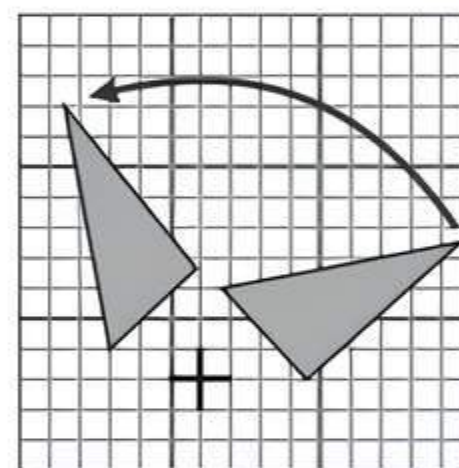
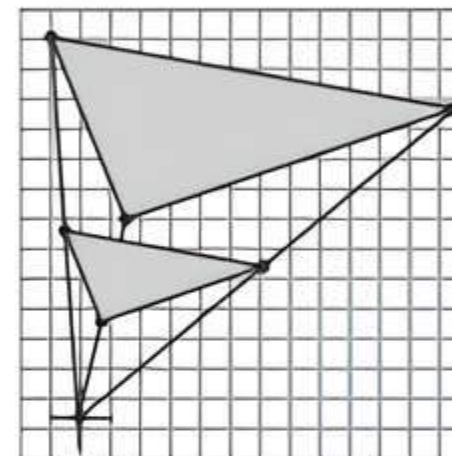
Case Study Based Question

Q 9. **Scale Factor:** A scale drawing of an object is of the same shape as the object but of a different dimension.

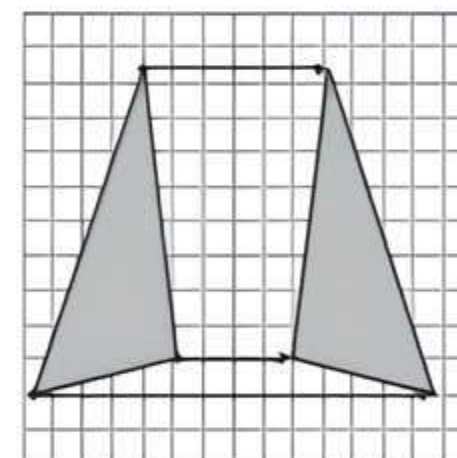
The scale of a drawing is a comparison of the length used on a drawing to the length it represents. The scale is written as a ratio.

Similar Figures: The ratio of two corresponding sides in similar figures is called the scale factor.

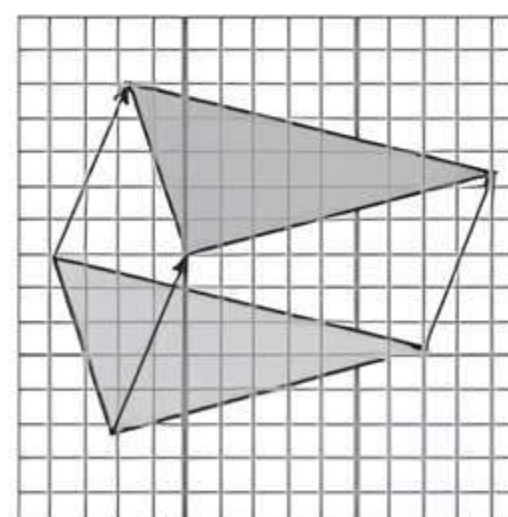
If one shape can become another using resizing then the shapes are similar.



Rotation or Turn



Reflection or Flip



Translation or Slide

Hence, two shapes are similar when one can become the other after a resize, flip, slide or turn. Based on the above information, solve the following questions:

- (i) A model of an aeroplane is made to a scale of 1 : 400. Find the length (in cm) of the model, if the length of the aeroplane is 40 m.



(ii) Find the length (in m) of the aeroplane if length of its model is 16 cm.

(iii) A $\triangle ABC$ has been enlarged by scale factor $m = 2.5$ to the $\triangle A'B'C'$. Find the length of $A'B'$, if AB is 6 cm.

Or

Find the length of $C'A'$, if $CA = 4$ cm.

Very Short Answer Type Questions

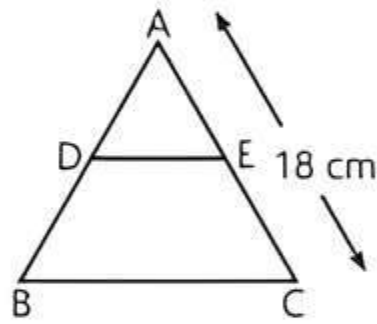
Q 10. If the corresponding altitudes of two similar triangles are in the ratio 3 : 5, then find the ratio of their corresponding sides.

Q 11. If in triangles ABC and DEF , $\frac{AB}{DE} = \frac{BC}{FD}$ and $\angle B = \angle D$, then these triangles will be similar by which criteria?

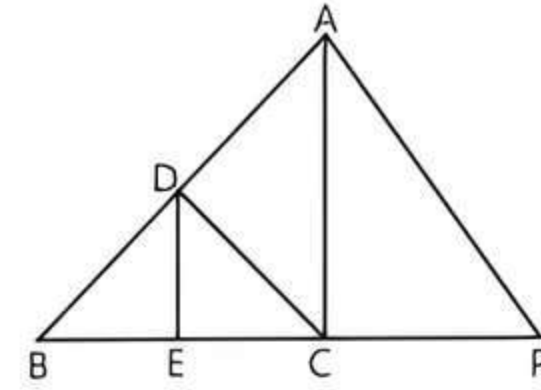
Short Answer Type-I Questions

Q 12. In the given figure, $DE \parallel BC$. If $\frac{AD}{DB} = \frac{2}{3}$ and

$AC = 18$ cm, find AE .

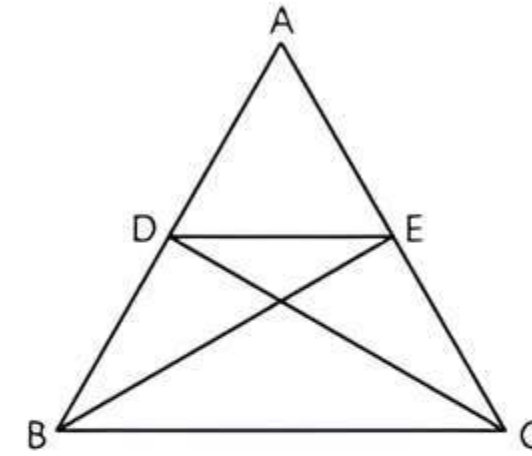


Q 13. In the given figure, $DE \parallel AC$ and $DC \parallel AP$. Prove that $\frac{BC}{CP} = \frac{BE}{EC}$.

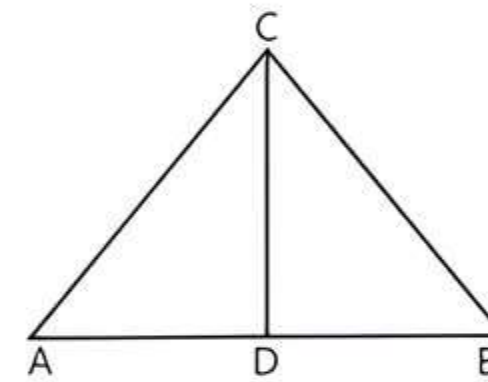


Short Answer Type-II Questions

Q 14. In the given figure, if $\triangle ABE \cong \triangle ACD$, prove that $\triangle ADE \sim \triangle ABC$.



Q 15. In the given figure, $\angle ACB = 90^\circ$ and $CD \perp AB$. Prove that $\frac{CB^2}{CA^2} = \frac{BD}{AD}$.



Long Answer Type Question

Q 16. Through the vertex D of a parallelogram $ABCD$, a line is drawn to intersect the sides BA and BC produced at E and F respectively. Prove that

$$\frac{DA}{AE} = \frac{FB}{BE} = \frac{FC}{CD}$$